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EVALUATION AND DEVELOPMENT OF TRANSMISSION LINE FAULT-LOCATING TECHNIQUES WHICH USE SINUSOIDAL STEADY-STATE INFORMATION

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INTRODUCTION

Fault-location algorithms which use steady-state data are analyzed and compared. The method of computing the apparent reactance seen from one end of the faulted line does not provide sufficient accuracy in general. A new method, described by Takagi [4], is much better, but requires assumptions to be made concerning the system external to the monitored line. A modification is offered, which provides a simple method of improving the performance of the Takagi algorithm when some system data are available. A two-end algorithm is presented, which requires no assumptions to be made on system parameters outside the monitored line. Its communications requirements are very modest. Manual or automatic means are suitable, and its implementation is practical in present-day technology [5].

DERIVATION OF ALGORITHMS

The voltages, currents and their subscripts are defined using Figure 1. The model system consists of two buses, S and R, linked by a single transmission line. The parts of the system to the left of bus S and to the right of bus R are repreresented by Thevenin equivalent circuits. The fault location is a fraction, m, of the total line length from S to R. The fractional distance is zero at bus S and one at bus R. The model considers only single-mode sinusoidal steady-state conditions. Concentration is placed on the goal of obtaining an algorithm which is little affected by fault resistance.

Reactive Component of Apparent Impedance

The technique used by reactance relays is to measure the apparent impedance of the line. Under fault conditions, the idea is that the apparent impedance should equal that of the line to the fault location, plus the fault impedance. Given that the transmission line is quite reactive, and that the fault impedance tends to be quite resistive, the reactive component of the apparent impedance is proportional to the distance to the fault. That is,

$$V_S = m\ Z_L\ I_S + R_F\ I_F \quad (1)$$

$$V_S/I_S = m\ Z_L + R_F\ I_F\ /\ I_S \quad (2)$$

$$\mathrm{Im}\ [V_S/I_S] = m + \mathrm{Im}[R_F\ I_F\ /\ I_S]\ /\ \mathrm{Im}[Z_L] \quad (3)$$

On the right-hand side, m is the fault location, expressed as a fraction of the entire line length, Z_L, and I_F is the fault current flowing through the fault resistance R_F. Thus, when the apparent reactance is used as the estimate of the fault location, an error results which depends on the angle between the fault current and the current from bus S. If that angle is zero, then the error is zero. The errors in this method are described in Warrington [1, 2] and in the references he cites. The error is affected by double-end feed, load impedance, power transmission angle, and angles of line and system impedances. The next section of this paper compares the errors of this traditional method with other techniques.

Method of Takagi et al [4], and an Improvement

In their 1982 paper, the authors describe a method which uses prefault current as well as postfault current and voltage. The approach is to find a quantity which, when multiplied by the fault-voltage term, yields a purely real result. Then, when the imaginary components are selected and compared, an estimate of the distance to the fault is obtained, which is independent of the fault resistance.
The key is to find a quantity which is locally observable, and which is proportional to the complex conjugate of the fault voltage. Consider conditions at the fault. The fault current is the sum of the currents from bus S and bus R. It may also be expressed as the sum of the superposition currents from buses S and R, which would flow if a superposition source of $-V_{F0}$ is placed at the fault location, and if all other sources are set to zero. The superposition current component from bus S is the difference betweent the postfault and prefault currents from bus S. That is,

$$I_S'' = I_{S0} - I_{SF} \qquad (4)$$

where the subscripts S0 and SF denote prefault and postfault conditions, respectively. Inasmuch as I_S'' is a part of the fault current, we would expect I_S'' and I_F to be in close phase relationship, especially when most of the fault current comes from the S side of the system. To aid algorithm development, and the comparisons of the next section, define the ratio of the fault current to the S-side superposition current component as:

$$A = I_F/I_S'' \quad \text{and} \quad T = \text{angle}\ (A) \qquad (5)$$

Then, multiply (3) by the complex conjugate of (5) solved for I_F^*:

$$V_S I_S''^{*} A^{*} = m\, Z_L I_S I_S''^{*} A^{*} + R_F I_F I_F^{*} \qquad (6)$$

When the imaginary part of (6) is taken, the fault-resistance term disappears, and the result may be solved for m:

$$m = \frac{\mathrm{Im}[V_S I_S''^{*} \exp(-jT)]}{\mathrm{Im}[Z_L I_S I_S''^{*} \exp(-jT)]} \qquad (7)$$

Takagi et al then sets T equal to zero, giving the final result:

$$m = \mathrm{Im}[V_S I_S''^{*}]/\mathrm{Im}[Z_L I_S I_S''^{*}] \qquad (8)$$

As will be seen, this technique offers substantial improvements over (3), in most cases. However, the estimation error of (8) is not always smaller than that of (3).

The angle T can be determined from the circuit model. It is the angle of the following expression:

$$I_F/I_S'' = 1 + [Z_{TS}/Z_L + m] / [Z_{TR}/Z_L + (1-m)] \qquad (9)$$

The angle T is zero if all impedances share the same angle.

The angle T is constant if Z_{TR} and Z_L have equal angles.

The improvement we offer is to use (7), and to provide an estimate of the angle, instead of setting it equal to zero. Two estimations are possible, and the choice depends on the knowledge and variability of system parameters.

1. Set T equal to a constant. This approach is particularly effective when the second condition above is met, i.e., when Z_{TR} and Z_L are of equal angle.

2. Determine T from a function of m, and iterate using (7) and the function. The best function is that implied by (9). Other choices include linear approximations or piecewise-linear approximations.

The Takagi paper [4] uses the alpha component of the superposition current, instead of the total current in the faulted phase, for single-line-to-ground faults. Use of the total current instead may not provide a current component which is as closely in phase with the fault current, if there is a substantial difference between the zero-component impedance angles on either side of the fault. Such could be the case if different grounding schemes were used on either end of the line.

Two-End Data Method

It is not possible, in general, to determine the location of a fault using sinusiodal steady-state information taken from only one end of the line, unless the impedances behind the buses as well as the line impedance are known. The method of Takagi and the improvement offered both use information previously ignored to improve the estimate of fault location using data from one line end only. Baker and Flechsig [3] showed that it is possible to locate faults using data from one end of the line, if a traveling-wave approach is taken, and if the characteristic impedances and propagation constants of the line are known. The only assumption their method includes is that a substantial impedance discontinuity exists at the bus where the observations are made.

It is possible to accurately locate faults using sinusoidal steady-state voltage and current information taken from both ends of the line, if the transmission line constants are known. Methods for short lines and long lines are described below.

There is no requirement for real-time communications. All that is needed is a record of the three voltages and three currents at each end of the line during the fault, and a record of at least one prefault current from each end. The data at each end of the line need to be measured as phasor quantities, but the relative phase of the phasor references used at each end is arbitrary.

This scheme was developed to offer an improved method of locating faults using digital protection systems at each end of a line, such as the prototype system described by the author and Jachinowski in [5]. The prototype system saves a record of data samples, taken at a rate of four samples per power system cycle. The record includes prefault data and postfault data for all three voltages and currents. The stored information is available, via a serial communications interface, for local or distant access, and the algorithms described below are capable of accurate fault-location computations using that stored information. Prefault current is assumed available for determining the relative angle between the phasor references.

Suppose that the postfault voltages and currents from bus S and from bus R are available at bus S. The voltage at the fault point can be written in terms of the S-end voltage and current, and in terms of the R-end voltage and current. When these are equated, the result is an equation which can be solved for m, in terms of the line impedance and the voltages and currents at both ends of the line.

The voltage at the fault point, V_F, computed using S-bus data, is:

$$V_F = V_S - mZ_L I_S \qquad (10)$$

The voltage at the fault point computed using R-bus data, is:

$$V_F = V_R - (1-m)Z_L I_R \qquad (11)$$

When these are equated so as to eliminate V_F, and solved for m, the result below is obtained:

$$m = [(V_S - V_R) + Z_L I_R] / [Z_L(I_S + I_R)] \qquad (12)$$

This equation gives a complex value for m, which represents the fractional distance to the fault point, when a real value is desired. The imaginary component accounts for some errors in the constants, e.g. the angle of the line impedance, or for some measurement error or noise in the data. We also expect errors for the same reasons in the real part as well. Let M be the true distance to the fault. Then, the complex value of the fault location, m, may be considered as follows:

$$m = M + P + jQ$$

where P + jQ represents the error in the measurement of M. The estimated fault location could be taken as the real part of m, or as the magnitude of m.

For small values of Q, there is little difference between the estimated, and computational efficiency favors using the real part.

The estimate given by the real part of (12) is indeterminate when no conduction takes place between buses S and R along the line. That is, under normal conditions, $V_S - V_R = I_R - Z_L$, and $I_S - I_R = 0$. These relationships provide a convenient means of checking the calibration of the equipment used to implement the algorithm.

The impedance at the fault point is estimated using:

$$Z_F = [V_S I_R + V_R I_S - Z_L I_R I_S] / (I_R + I_S)^2 \tag{13}$$

The phase-referencing of measurements taken at both ends of the line is easily accomplished by observing the prefault currents. For example, the short-line approximation assumes the currents at both ends of the line are the same under normal, unfaulted, conditions. That is, the prefault condition is:

$$I_R = -I_S \tag{14}$$

Thus, observation of the prefault current yields the phase relationship between the 'clocks' at the two substations against which all the voltages and currents are compared.

Where the short line approximation is unjustified, e.g. with lines longer than about 50 miles, the derivation may be repeated using distributed-parameter equations. Equations (10) and (11) are replaced by:

$$V_F = V_S \cosh gmL - Z_S I_S \sinh gmL \tag{15}$$

$$V_F = V_R \cosh g(1-m)L - Z_S I_R \sinh g(1-m)L \tag{16}$$

where g = propagation constant
and L = line length.

The solution obtained using these equations and the first-order approximations for the exponentials in the hyperbolic functions is:

$$m=[(V_S-V_R)-Z_S gLI_R]/[Z_S gL(I_S+I_R)-(V_S-V_R)(gL)-(V_R+Z_S I_R)(gL)^2] \tag{17}$$

where $Z_S gL = Z_L$

A S-bus estimate of prefault I_R is needed to synchronize the clocks. Using the same approximations, the estimate is:

$$I_{RS} = V_S[gL/Z_S] - I_S \quad (18)$$

where the RS subscript denotes the estimate of the R-end quantity at the S-bus.

The accuracy of this method depends mainly on the accuracy of the computed line parameters at system frequency, and the accuracy of the measurements of the voltages and currents. Normalized factors for the sensitivity of m to a change in the voltages, currents or line impedance are given below, and were determined using the short-line model. These factors may be used to estimate the percent change in m expected for a given percent change or error in any one of the aforementioned parameters.

$$\text{Sensitivity of m on } Z_L = -[1 + Z_L I_R/(V_S - V_R)]^{-1} \quad (19)$$

$$\text{Sensitivity of m on } V_S = V_S/[V_S - V_R + Z_L I_R] \quad (20)$$

$$\text{Sensitivity of m on } V_R = -V_R/[V_S - V_R + Z_L I_R] \quad (21)$$

$$\text{Sensitivity of m on } I_R = [Z_L I_R I_S - I_R(V_S - V_R)]/\{[(V_S - V_R) + Z_L I_R][I_S - I_R]\} \quad (22)$$

$$\text{Sensitivity of m on } I_S = I_S/(I_S - I_R) \quad (23)$$

COMPARISONS OF PERFORMANCE

The reactance algorithm and the Takagi algorithm were tested using a simulation of the model of Fig. 1. The simulation demonstrates the important performance improvement that the Takagi algorithm offers in a wide variety of conditions. The simulation also demonstrates that performance benefits from a better estimate of the angle between the fault current and the fault current component from the bus where the fault location is attempted.

The simulation results demonstrate that conditions exist for which the reactance and Takagi algorithm accuracies may be considered insufficient. The recourse is to provide additional system parameter information via the angle T in the modified Takagi algorithm, or to use information from both ends of the line.

Table 1 provides a list of the parameters used in six test cases.

CASE	FIGURE	Z_{TS}	Z_L	Z_{TR}	R_F	COMMENTS
1	2,3	0.01+j0.1	0.10+j0.5	0.01+j0.1	0.04	bus X/R=10,line X/R=5
2	4,5	0.01+j0.1	0.10+j0.5	0.02+j0.1	0.04	line & bus R X/R=5
3	6	0.02+j0.1	0.10+j0.5	0.01+j0.1	0.04	line & bus S X/R=5
4	7,8	0.02+j0.1	0.25+j0.5	0.05+j0.1	0.04	line & bus R X/R=2
5	9	0.04+j0.1	0.25+j0.5	0.04+j0.1	0.04	bus S&R X/R = 2.5
6	10	0.04+j0.1	0.25+j0.5	0.20+j0.5	0.04	weak source at R

Table 1. Model System Parameters for Simulations

The bus impedances are nominally 0.1 pu, and the line impedance is nominally 0.5 pu. In all cases, a fault resistance of 0.04 pu is used; however, larger values are additionally used in the plotted data of the figures. For the first three cases, the X/R ratio of the line is held at 5, and for the remaining three cases, the X/R ration is held at 2.5. For cases 2 and 4, the line and R-bus X/R ratios are equal. The R-bus impedance of case 6 is five times as large as in the other five cases. This was done to observe the diminishing error as the amount of infeed from the far end is limited by the impedance behind the bus.

The angle between the Thevenin sources is either +30 or -30 degrees. All but three of the figures display data for both conditions. The curves are labeled + or - to indicate which way power flow was occuring. In the remaining three figures (Figs. 3,5, and 8), the angle of -30 degrees alone is used.

Each figure displays the error in the computed fault location as a function of actual fault location. The 'percent error' computed is the fault location error shown as a fraction of the total line length. Thus an error of 5% in a fault location computation for a fault actually 50% away from S-bus indicates that the computed location is 55%.

In all figures except Fig. 6, the solid lines are plots of the reactance algorithm errors, and the dotted lines are plots of the Takagi algorithm error or the modified Takagi algorithm error.

The reactance algorithm errors displayed in Fig. 2 (case 1) clearly show the underreaching for power flow R to S (power angle = -30 deg.), and the overreaching for power flow S to R. The improvement offered by the Takagi algorithm is dramatic; however, far-end errors develop suddenly. The error remains less than one percent for up to about 80% of the line under case 1 conditions shown in this figure.

The effect of increasing the fault resistance is shown in Fig. 3 for the same case, but for a power angle of -30 degrees only. The R_F = 0.04 pu curve is the same as the -30 degree curve of Fig. 2. The effect of increasing the fault resistance by a factor of three is to increase the near-end error from nearly zero to about 0.5%, and to cause the one-percent error distance to move from around 80% to about 65%.

Case 2 shows the improvement which results if the Thevenin impedance of the system beyond R-bus has the same angle as the transmission line. See Fig. 4. The angles were made equal by doubling the resistive component of Z_{TR} from the value used in case 1. The reactance algorithm results change very slightly from the case 1 results. The Takagi results improve, since the angle T under the conditions of case 2 is a constant: T = 0.8 degrees. On the other hand, for case 1, T ranges from about 0.8 near the S-bus to about 4.5 near the R-bus.

Fig. 5 shows case 2 results for several different fault resistance values for the Takagi algorithm (upper three curves) and the modified Takagi algorithm. A constant value of T = 1 was used for the modified Takagi algorithm. These results demonstrate that an assumed value of T other than zero can provide improved performance. Of course, had an exact value of T been used, the error would have been zero.

Case 3 demonstrates that matching the impedance angle between the line and the S-bus has little effect on the reactance algorithm or on the Takagi algorithm. The error is larger for case 3 than for case 1 for far end faults. The effect on close-in fault location accuracy is negligible. This can be seen by comparing Figs. 2 and 6.

For cases 4,5, and 6, the X/R ratio of the transmission line is X/R=2.

For case 4, the X/R ratios of the line and the R-bus Thevenin impedance are equal, which implies that the angle T is constant. In fact, T= 2 degrees. Fig. 7 illustrates that the Takagi algorithm error reaches one percent at 70% of the line length, when power flow is towards bus S. The modified Takagi algorithm offers substantial improvement, if a good estimate of T is available from system data. In Fig. 8, the errors for the modified Takagi algorithm are plotted for five different values of T. The T=0 curve is the same as the Takagi algorithm error curve of Fig. 7 for a -30 degree power angle. The family of curves of Fig. 8 clearly show the sensitivity of the Takagi algorithm on T. If the curves were replotted with a larger vaule of fault resistance, they would appear further spread out in the vertical dirction.

For case 5, X/R for the Thevenin impedances is 2.5, and for the line is 2.0. Even though X/R is low, the R-bus value is close to the line value, so the errors produced by the Takagi algorithm are not excessive.

The R-bus impedance for case 6 is five times that of case 5, thereby reducing the amount of infeed from bus R. The effect is an expected substantial reduction in the far-end error in the Takagi algorithm.

Cases 1 and 5 (Figs. 2,3, and 9) show that the angle T between the fault current and its superposition component from S-bus is zero when the fault is at the impedance midpoint of the system, since the errors of the Takagi algorithm are zero at the line midpoint. Note that the two bus impedances are identical in these cases.

Several conclusions can be drawn.

1. Although a reactance-based algorithm provides sufficient accuracy for many protective relaying applications, it is not accurate enough, in general, for use as a fault-locating algorithm.

2. The algorithm offered by Takagi offers substantial improvement over the reactance-based method, by accounting for load flow.

3. The modification to the Takagi algorithm offered in this paper, namely, selecting values of T other than zero, can offer performance improvements. If one can provide a relationship between T and the fault location, then this is one means of advantageously using additional information about the power system to locate faults.

4. The Takagi algorithm is very sensitive to variations in T, as can be seen in Fig. 8, and can be shown using sensitivity factors.5. To be confident that the Takagi algorithm provides good performance, a user must be sure the application is one where the system is nearly homogeneous.

The two-end method described in the previous section does not require or assume values of the system parameters external to the line being monitored. Thus, if its performance were measured using the modeling technique used to evaluate the reactance and Takagi algorithms, then the result would be zero error under all conditions. The accuracy of the two-end method is limited by the accuracy of the line model used in a particular implementation (e.g. short line vs distributed parameter), by the model used to describe the zero-sequence mode, by the accuracy to which the parameters of the models can be computed or determined, and by the accuracy of the measurement system. All of these factors also affect the reactance and Takagi algorithms.

PRACTICAL CONSIDERATIONS

Manual or automatic comparison of the data from both ends of the line provides an accurate means of locating faults. Comparison under normal conditions is a check on the integrity of the terminal equipment, which, if done regularly, offers an enhancement to the potential availability of that equipment. If the data are not available from both ends, then the Takagi algorithm or its modified version can be used, with less assurance of accuracy, especially for far-end faults. Modern digital equipment, such as that described in [4,5], provides the capabilities for implementing any of the schemes described in this paper.

CONCLUSIONS

1. The reactance algorithm is essentially useless for accurate location of resistive faults when two-end feed is possible.

2. The Takagi algorithm, under many system conditions, offers good performance using data from only one end of the line for faults between the near end to about 75%-85% of the line.

3. The modified Takagi algorithm offers enhanced performance, if systems information is available to estimate the angle T between the fault current and the fault current component from the measurement end of the transmission line.

4. The accuracy of the Takagi algorithm is very dependent on the system conditions that affect the angle T. That is, the error is very sensitive to changes in T. Accuracy can be guaranteed only if it is known that the system is nearly homogeneous.

5. The two-end algorithm provides an alternative which does not require any assumptions on the system outside the monitored line.

6. The requirements for communications of data from the line ends to a common point are very modest. Automatic or manual means can be used, and there is no requirement for time synchronization based on the communications channel properties.

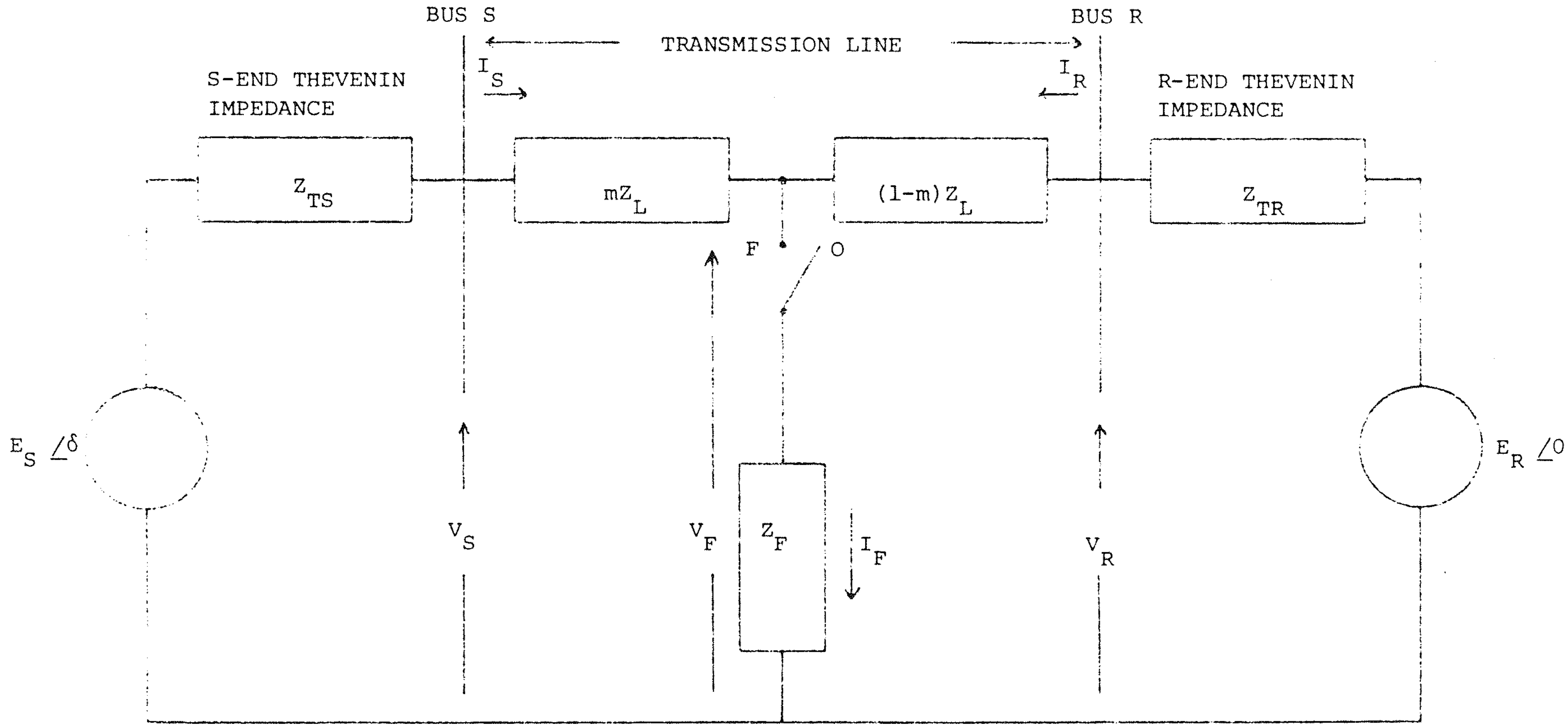

Figure 1. Circuit Model of Two-Bus System

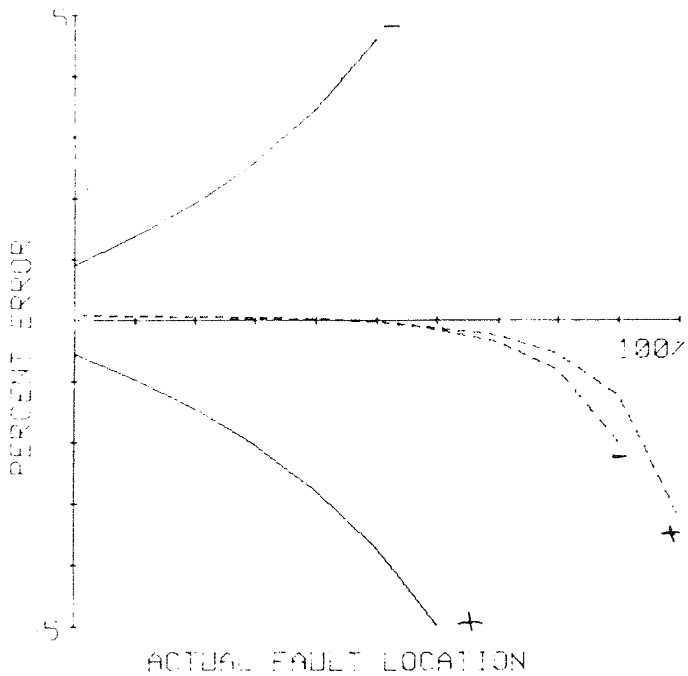

Figure 2. Case 1. Reactance (Solid) and Takagi (Dotted) Algorithm Errors for Power Angles of ± 30 degrees.

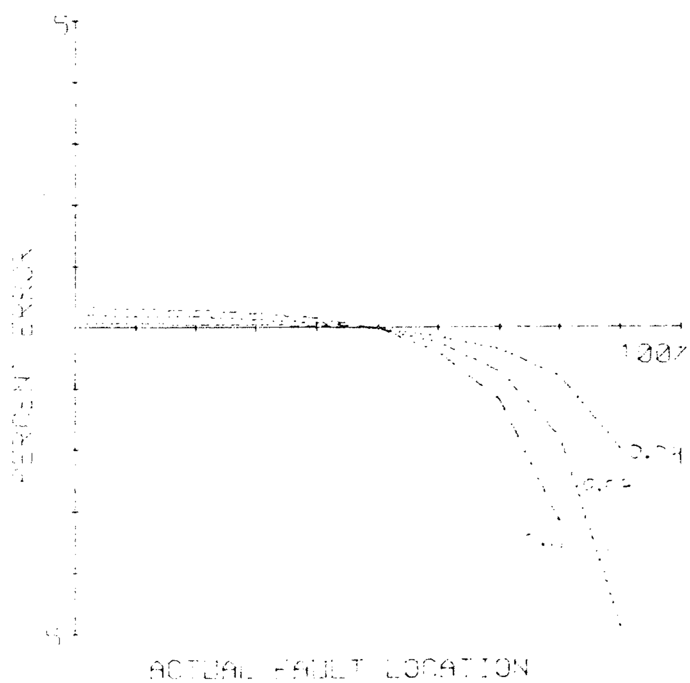

Figure 3. Case 1. Takagi Algorithm Error for Three Values of Fault Resistance.

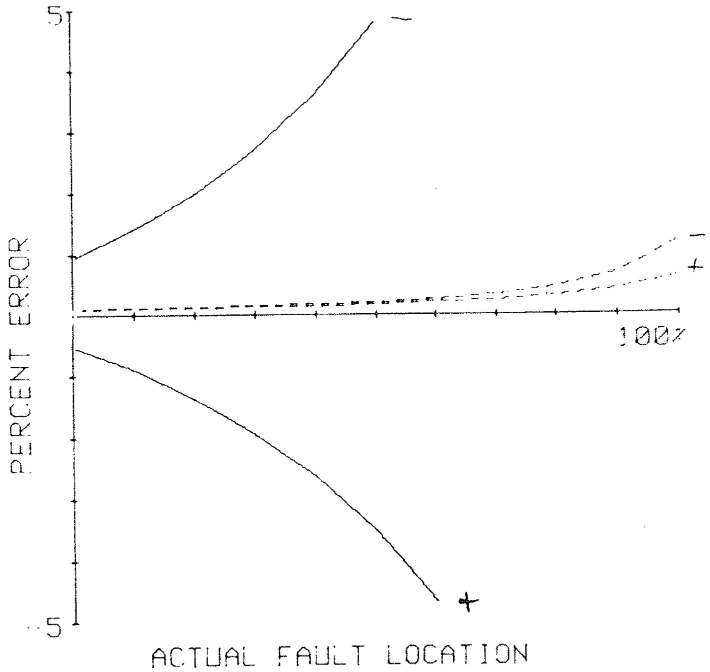

Figure 4. Case 2. Reactance (Solid) and Takagi (Dotted) Algorithm Errors When Line and Bus R X/R = 5.

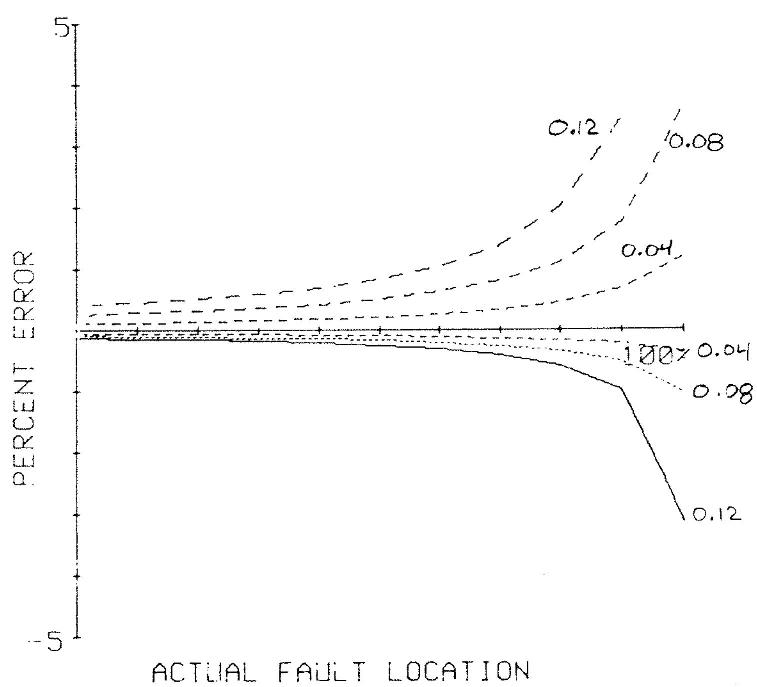

Figure 5. Case 2. Takagi (Upper 3 Curves) and Modified Takagi (T=1 degree) (Lower 3 Curves) Errors.

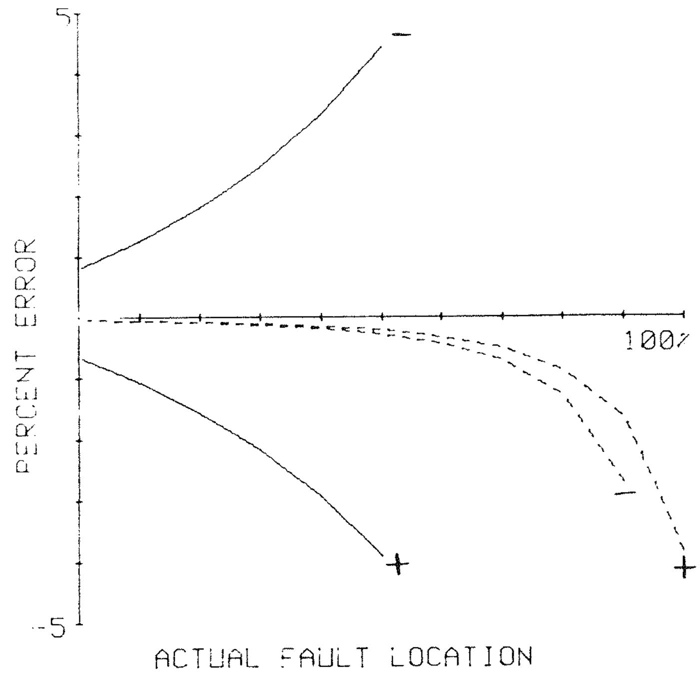

Figure 6. Case 3. Reactance (Solid) and Takagi (Dotted) Algorithm Errors When Line and Bus S X/R = 5.

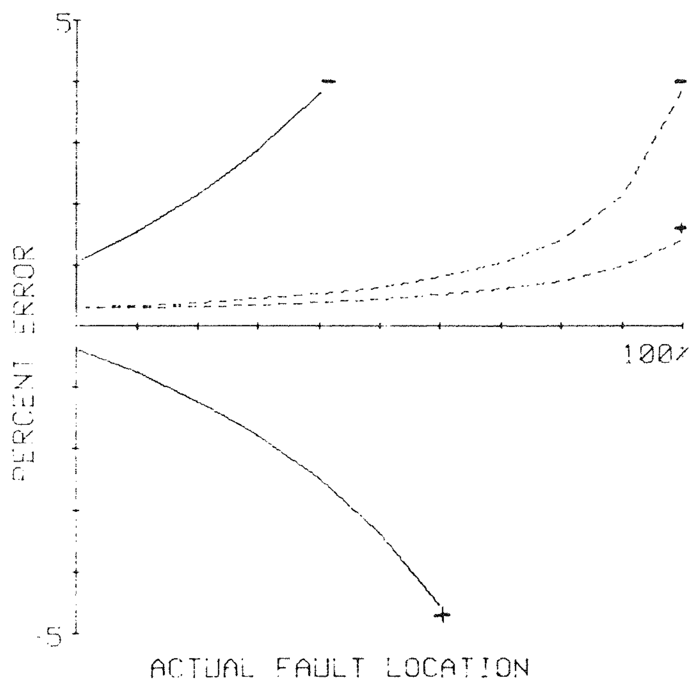

Figure 7. Case 4. Reactance (Solid) and Takagi (Dotted) Algorithm Errors When Line and Bus R X/R = 2.

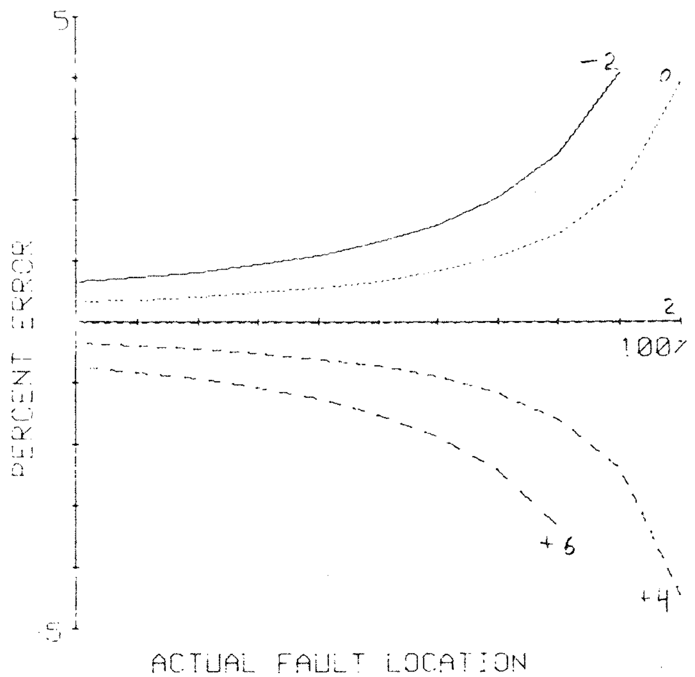

Figure 8. Case 4. Modified Takagi Algorithm Error for Five Values of Angle T.

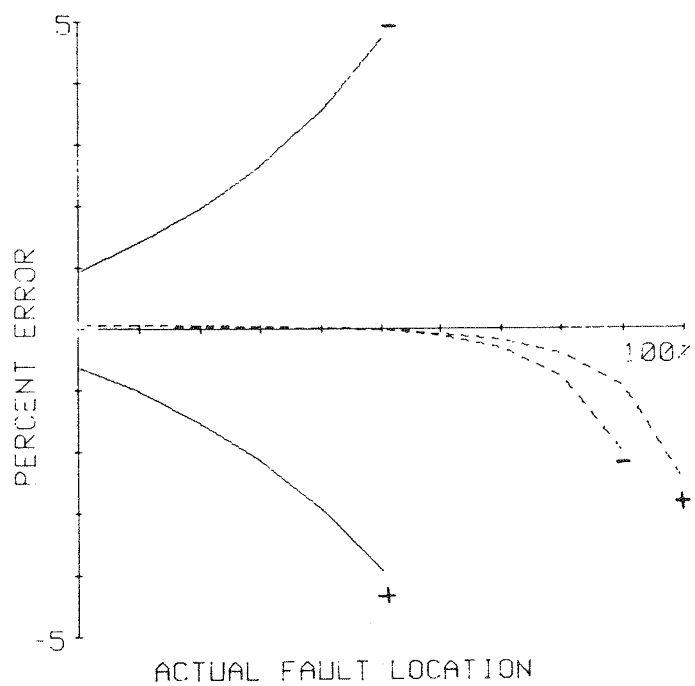

Figure 9. Case 5. Reactance (Solid) and Takagi (Dotted) Algorithm Errors for Bus S and R X/R = 2.5.

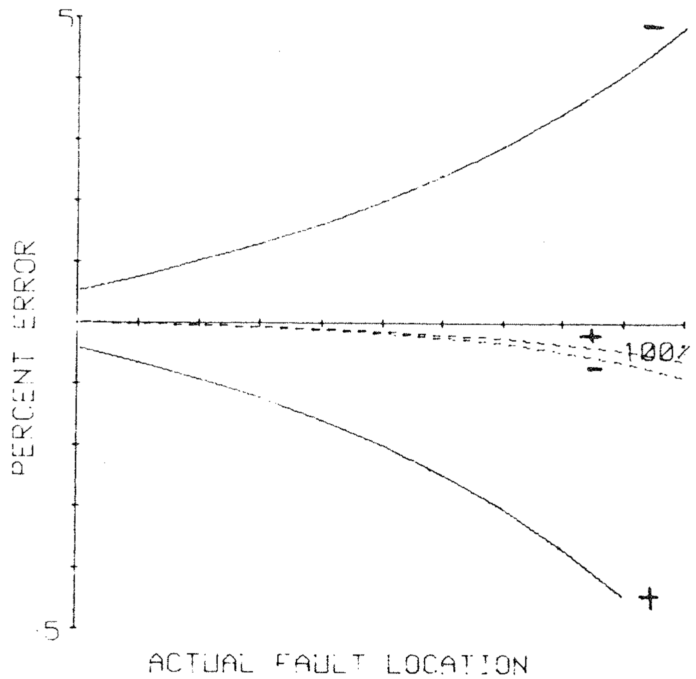

Figure 10. Case 6. Reactance (Solid) and Takagi (Dotted) Algorithm Errors for Weak Infeed From Bus R.

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