Distance Elements: Linking Theory With Testing

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***Abstract*—Understanding and testing distance elements require linking the theory related to their design to the fault voltages and currents used during tests. This paper discusses the theory, deriving the characteristics of phase and ground distance elements with a focus on three in particular. The derivation will introduce certain concepts, including the impedance plane, source impedance, forward fault impedance, reverse fault impedance, fault resistance, and load flow behavior. The discussion is based on the use of symmetrical components to understand the design, behavior, and characteristics of distance elements. This paper also reviews some testing procedural errors. A confident understanding of the theory allows for an easier understanding of the testing results obtained.**

I. Introduction

Distance units are the main protection algorithms implemented in most line protection schemes. Even with line differential schemes, distance schemes are used as backup, keeping these elements in the protective relaying inventory.

While different distance elements have been proposed according to the requirements and preferences of manufacturers, there are concepts that should be linked from the academic analysis to the practical testing of these elements. Knowing the theory that derives the characteristics of the element makes it easier to understand the behavior while testing the element or comprehending its operation in the field.

Distance units have been traditionally categorized in phase and ground units. They have been designed to detect power system faults that do not involve the ground return (3I0) for phase faults and those that involve the ground return for ground faults. Phase-to-phase-to-ground faults are a design choice for manufacturers; generally, these faults will be associated with phase fault detection.

With ten fault types possible, determining the impedance to the fault is not a simple measurement of voltage and current [1][2][3]. In traditional line protection schemes, six impedance units are used, usually identified as AG, BG, CG, AB, BC, and CA elements. These six elements constitute a protection zone and should detect all of the ten fault types possible.

Distance elements are used in line protection schemes for their definite reach. Simple overcurrent protection lacks this property, and its reach is highly dependent on the source impedance. A definite reach is highly desired in distance elements.

In general, protective relaying elements are implemented by comparing thresholds to measured and/or combined power system quantities. While early electromechanical distance relays used actual magnitude comparisons of the voltage-produced restraining torque to the current-produced operating torque [4], distance elements have been preferably implemented by comparing the phase relationship of two vectors. The comparison is done with a phase comparator, which can be implemented with an electromechanical unit (a two-phase motor, for example), a solid-state coincidence timer (measuring a quarter-cycle coincidence between square waves), or numerical comparator [2][3][5]. Regardless of the technology, the threshold condition of a phase comparator is described by (1).

$$\arg\left(\frac{S1}{S2}\right) = \pm 90° \quad (1)$$

The quantities S1 and S2 are phasor quantities, inputs to the phase comparator. S1 is normally called the operating quantity, and S2 is the polarizing quantity or reference. For distance elements, S1 will normally be the assumed voltage drop (V – Zc I) across the element's replica impedance Zc. Voltage V is the restraining voltage, and (Zc I) is the operating quantity that overcomes the restraining voltage. To compare whether (Zc I) is larger than V with a phase comparator, a reference quantity is required (S2).

S2 is the reference and must be a stable quantity (no significant phase change) during normal and faulted power system conditions. The choice of S2 may differ from manufacturer to manufacturer, and benefits and drawbacks have been described in literature [2].

Fig. 1 shows the theoretical operation of S1 and S2 for faults in the reverse direction (Fig. 1a), in zone (Fig. 1b), and out of zone (Fig. 1c) for a distance unit. S2 should be a solid reference that should not change drastically during fault conditions.

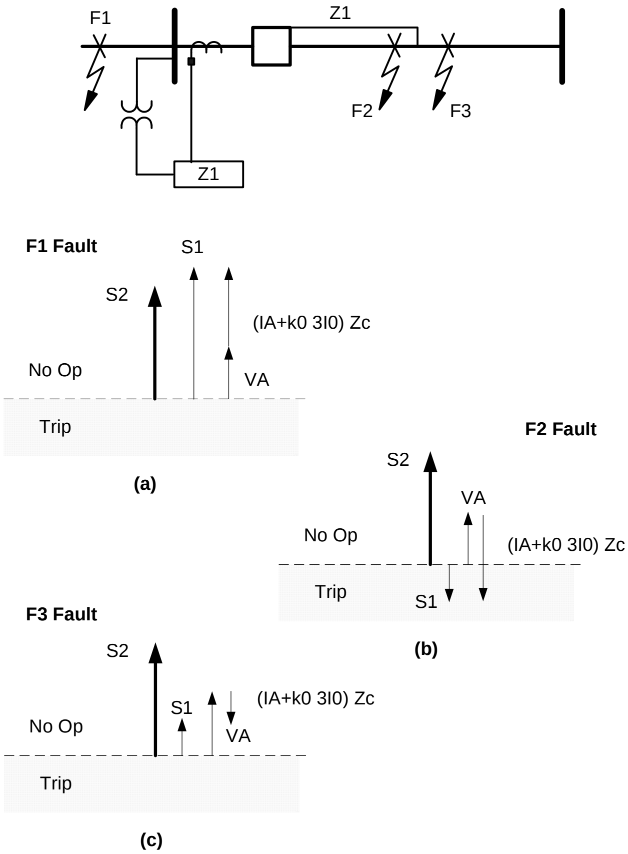

Fig. 1. Distance Relay Operation

Protective relaying theory is heavily based on the theory of symmetrical components. The derivation of distance-unit characteristics is no exception. In fact, it is the theory that uses symmetrical components in the most elegant and ingenious way.

II. Describing a Circle

Mho distance elements are circles on an impedance plane. An equation describing a circle should lead to the description of these distance units. Equation (2) is a vector equation of a circle [3].

$$\arg\left(\frac{S1}{S2}\right) = \arg\left(\frac{\vec{Z} - \bar{a}}{\vec{Z} - \bar{b}}\right) = \pm 90° \tag{2}$$

Equation (2), which is an extension of (1), can be readily visualized in Fig. 2 for an arbitrary set of *a* and *b* vectors.

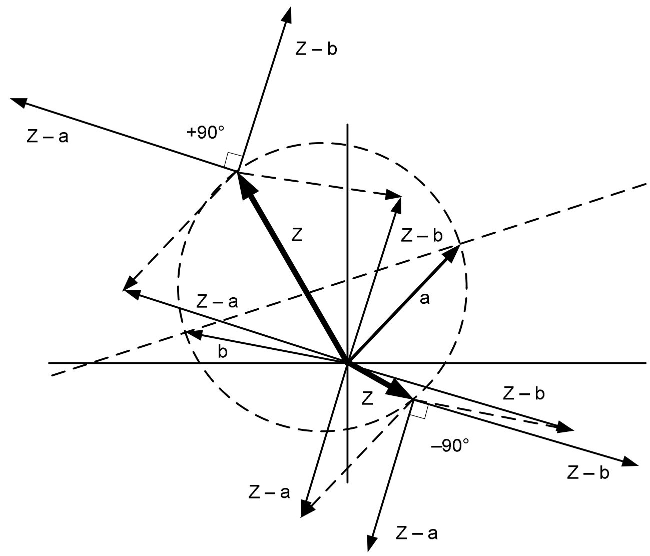

Fig. 2. Circle Defined by Arbitrary *a* and *b* Vectors on the Z Plane

Fig. 2 illustrates a circle and the graphical proof of (2). One-half of the circle satisfies the +90° requirement; the other half satisfies the –90° requirement. The circle is split by the line connecting the tips of *a* and *b*.

An academic and important fact to constantly remember is that the plot in Fig. 2 represents the circle on the Z plane. Keeping this concept in mind avoids mixing other or similar results that are based on different planes.

A simple and generic example of a distance element is used to demonstrate (2). This element is a self-polarized distance unit, where:

$$S1 = V - Zc\,I \tag{3}$$

$$S2 = V \tag{4}$$

This distance unit is called self-polarized because the polarizing quantity (S2) is the same as the restraining voltage. Substituting the S1 and S2 values of (3) and (4) into (2) yields:

$$\begin{aligned} \arg\left(\frac{S1}{S2}\right) &= \arg\left(\frac{V - Zc\,I}{V}\right) = \pm 90° \\ \arg\left(\frac{V/I - Zc}{V/I}\right) &= \arg\left(\frac{Z - Zc}{Z}\right) = \pm 90° \end{aligned} \tag{5}$$

In (5), the *a* and *b* vectors can be easily recognized as:

$$a = Zc \tag{6}$$

$$b = 0 \tag{7}$$

The plot of this characteristic is shown in Fig. 3.

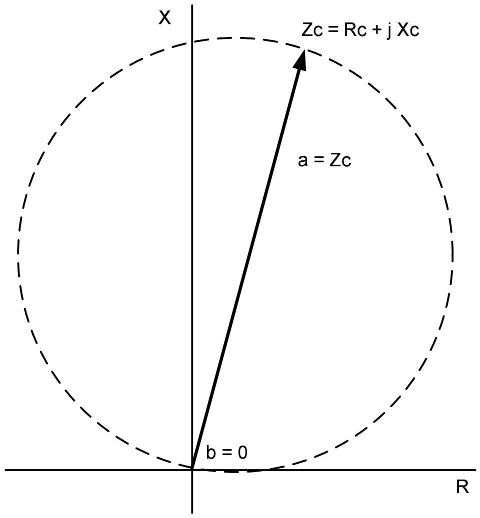

Fig. 3. Self-Polarized Distance Element on the Z Plane

III. Analysis of Two Mho Distance Elements

To support this discussion, two examples of mho distance elements are used: a ground distance unit and a phase distance unit. Both units polarized with positive-sequence voltage illustrate the proper development of distance element characteristics. The technique is also applicable to other elements found in the industry [3]. The analysis will review the AG and BC distance units, which are traditionally the most illustrative because they use Phase A symmetrical components. The other units are derived in a similar way and are an extension of these distance units.

A. Positive-Sequence Polarized AG Ground Distance Unit (Forward Characteristics)

This ground distance unit is designed with:

$$S1 = VRA - Zc1\left(IRA - kc0\ 3IR0\right) \tag{8}$$

$$S2 = VA1 \tag{9}$$

where:

$$kc0 = \frac{Zc0 - Zc1}{3Zc1} \tag{10}$$

Zc1 and Zc0 are the relay settings. Zc1 is the reach of the ground distance element, and Zc0 is implicitly defined in the relay settings when kc0 is adjusted for the particular application. These constants are to be visualized as the replica impedances of the line to be protected.

Fig. 4 illustrates the symmetrical component network interconnection of a source (with its own positive- and zero-sequence impedance) and an impedance located in the forward direction to the distance element location. The distance element measurements are denoted by R; these are the voltages and currents that the unit is measuring. The direction of the measurement is clearly shown with the current orientations.

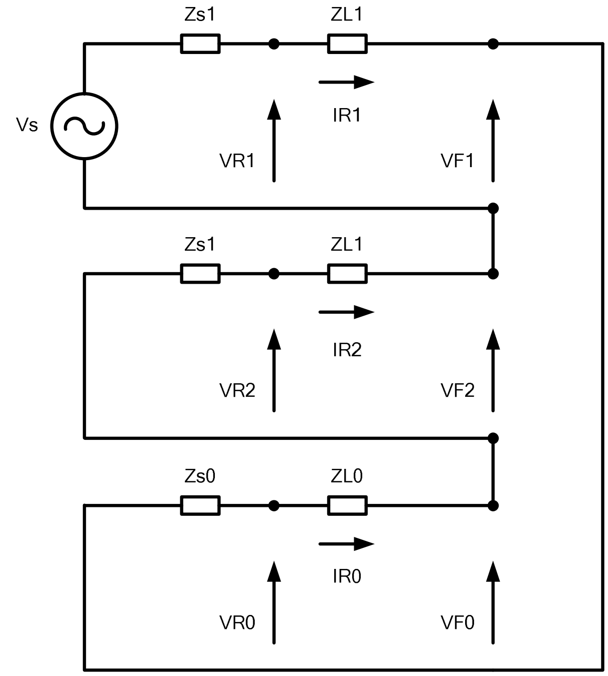

Fig. 4. Symmetrical Component Network for a Forward AG Fault

The impedance plane for which the analysis will be performed is the positive-sequence impedance in the forward direction. The analysis will find the ZL1 impedances for which the unit will operate. Interestingly, the following identity will allow for an easier analysis:

$$ZL1 = \frac{VR1 - VF1}{IR1} \tag{11}$$

Returning to (8), simplify to find a more useful expression for S1:

$$\begin{aligned} S1 &= \left(VR1 + VR2 + VR0\right) \\ &\quad - Zc1\left(IR1 + IR2 + IR0 - kc0\ 3IR0\right) \\ &= \left(VR1 - VF1\right) + VF1 + VR2 + VR0 \\ &\quad - \left(Zc1\,IR1 + Zc1\,IR2 + Zc0\,IR0\right) \end{aligned} \tag{12}$$

using:

$$\begin{aligned} VF1 &= -VF2 - VF0 \\ &= -\left(VR2 - ZL1\,IR2\right) - \left(VR0 - ZL0\,IR0\right) \end{aligned} \tag{13}$$

and:

$$IR1 = IR2 = IR0 \tag{14}$$

Next, simplify (12):

$$\begin{aligned} S1 &= \left(VR1 - VF1\right) + ZL1\,IR1 + ZL0\,IR1 \\ &\quad - Zc1\,IR1\left(2 + \frac{Zc0}{Zc1}\right) \end{aligned} \tag{15}$$

$$\frac{S1}{IR1} = ZL1\left(2 + \frac{ZL0}{ZL1}\right) - Zc1\left(2 + \frac{Zc0}{Zc1}\right) \tag{16}$$

Equation (16) is important in the analysis and is expressed in terms of ZL1, which is the impedance on which plane the characteristic will be derived.

Returning to (9), simplify to find a more useful expression for S2:

$$\begin{aligned} S2 &= (VR1 - VF1) + VF1 \\ &= (VR1 - VF1) - VF2 - VF0 \\ &= (VR1 - VF1) + (Zs1 + ZL1)IR2 \\ &\quad + (Zs0 + ZL0)IR0 \end{aligned} \tag{17}$$

Using (13) and (14), (17) simplifies to:

$$\frac{S2}{IR1} = ZL1\left(2 + \frac{ZL0}{ZL1}\right) + Zs1\left(1 + \frac{Zs0}{Zs1}\right) \tag{18}$$

The results in (16) and (18) can be used in (2) to find the *a* and *b* vectors:

$$a = Zc1\left[\frac{\left(2 + \frac{Zc0}{Zc1}\right)}{\left(2 + \frac{ZL0}{ZL1}\right)}\right] \tag{19}$$

$$b = -Zs1\left[\frac{\left(1 + \frac{Zs0}{Zs1}\right)}{\left(2 + \frac{ZL0}{ZL1}\right)}\right] \tag{20}$$

The impedance ratio of Zc1 and Zc0, via kc0 in (10), should equate the line impedance ratio of ZL1 and ZL0, because these are the replica impedances. Therefore, (19) is further simplified to:

$$a = Zc1 \tag{21}$$

Fig. 5 illustrates the derived characteristic on the ZL1 plane. Any ZL1 impedance (with a characteristic ZL0/ZL1 impedance ratio) inside the circle indicates an operating condition.

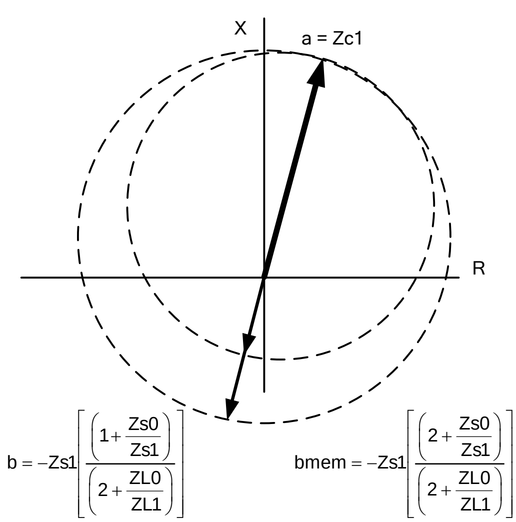

Fig. 5. Forward Fault Characteristic on the ZL1 Plane

Fig. 5 illustrates a very important concept—the fact that these impedances are all in the forward direction, as derived from the symmetrical component network interconnection in Fig. 4. A negative impedance (ZL1 = –jX), for example, does not mean the fault is in the reverse direction; it means that ZL1 is a capacitor in the forward direction.

To contrast the forward fault characteristics of distance elements, the reverse fault characteristics can be derived.

B. Positive-Sequence Polarized AG Ground Distance Unit (Reverse Characteristics)

The symmetrical component network connection shown in Fig. 6 is the network used to derive the reverse fault characteristics for the AG element described by S1 and S2 in (8) and (9). The impedance ZL′ is of interest, and the reverse fault characteristics of this distance unit will be plotted on the ZL1′ plane. Notice again the directions of the distance-unit current measurements are denoted by the current orientations.

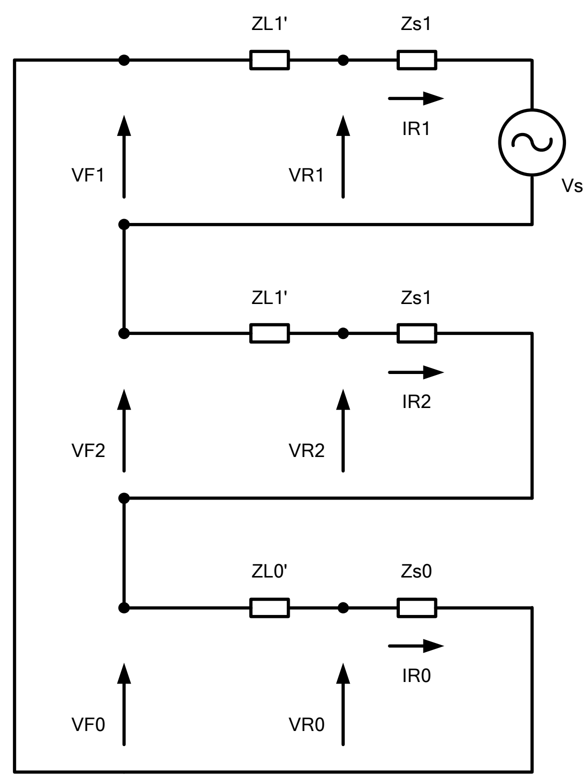

Fig. 6. Symmetrical Component Network for a Reverse AG Fault

Returning to (8), simplify to find a more useful expression for S1:

$$\begin{aligned} S1 &= (VR1 + VR2 + VR0) \\ &\quad - Zc1(IR1 + IR2 + IR0 - kc0\,3IR0) \\ &= (VR1 - VF1) + VF1 + VR2 + VR0 \\ &\quad - (Zc1\,IR1 + Zc1\,IR2 + Zc0\,IR0) \end{aligned} \tag{22}$$

using:

$$\begin{aligned} VF1 &= -VF2 - VF0 \\ &= -(VR2 + ZL1'IR1) - (VR0 + ZL0'IR0) \end{aligned} \tag{23}$$

Equation (22) can be simplified to:

$$\begin{aligned} S1 &= (VR1 - VF1) - ZL1'IR1 - ZL0'IR1 \\ &\quad - Zc1\,IR1\left(2 + \frac{Zc0}{Zc1}\right) \end{aligned} \tag{24}$$

using:

$$Zl1' = \frac{VR1 - VF1}{(-IR1)} \tag{25}$$

$$\frac{S1}{(-IR1)} = ZL1'\left(2 + \frac{ZL0'}{ZL1'}\right) + Zc1\left(2 + \frac{Zc0}{Zc1}\right) \tag{26}$$

Returning to (9), simplify to find a more useful expression for S2:

$$\begin{aligned} S2 &= (VR1 - VF1) + VF1 \\ &= (VR1 - VF1) - VF2 - VF0 \\ &= (VR1 - VF1) - (Zs1 + ZL1)IR1 \\ &\quad - (Zs0 + ZL0)IR1 \end{aligned} \tag{27}$$

$$\frac{S2}{-IR1} = ZL1'\left(2 + \frac{ZL0}{ZL1}\right) + Zs1\left(1 + \frac{Zs0}{Zs1}\right) \tag{28}$$

The results in (26) and (28) can be used in (2) to find the *a* and *b* vectors:

$$a = -Zc1\left[\frac{\left(2 + \frac{Zc0}{Zc1}\right)}{\left(2 + \frac{ZL0'}{ZL1'}\right)}\right] \tag{29}$$

$$b = -Zs1\left[\frac{\left(1 + \frac{Zs0}{Zs1}\right)}{\left(2 + \frac{ZL0'}{ZL1'}\right)}\right] \tag{30}$$

The ground element is mostly directional for all reactive impedances and may encounter problems if capacitances are located in the reverse direction. Fig. 7 illustrates that there are impedances on the ZL1′ plane (mainly capacitive and with negative resistance values) for which the element will operate. This is a desirable distance element characteristic. For security purposes, most manufacturers will opt to supervise the operation of distance elements with directional elements to avoid unwanted operations if the fault impedances in the reverse direction fall inside the characteristic, as shown in Fig. 7.

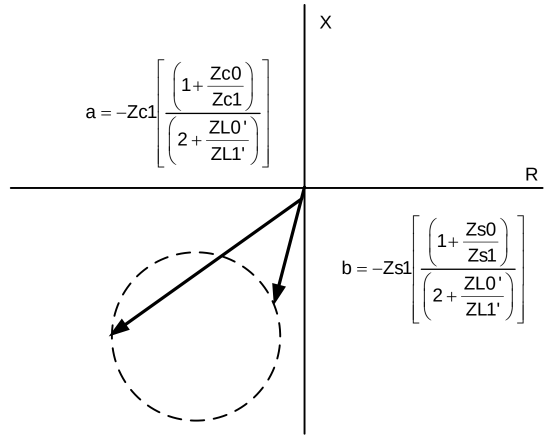

Fig. 7. Reverse Fault Characteristic on the ZL1′ Plane

C. Positive-Sequence Polarized BC Phase Distance Unit (Forward Characteristics)

The symmetrical component network connection shown in Fig. 8 is the network used to derive the forward fault characteristic for the BC element described in (31) and (32).

This phase distance unit is defined by:

$$S1 = (VRB - VRC) - Zc1(IRB - IRC) \tag{31}$$

$$S2 = VB1 - VC1 \tag{32}$$

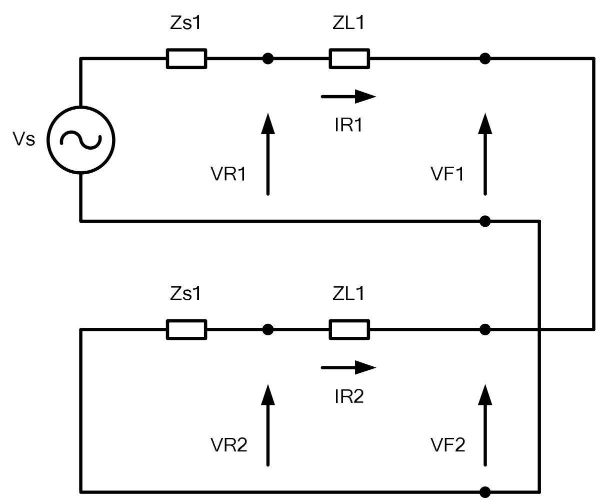

Fig. 8. Symmetrical Component Network for a Forward BC Fault

Following a similar procedure for S1 as that described in Section III, Subsection A:

$$\begin{aligned} S1 &= \left(a^2 - a\right)\left[\left(VR1 - VR2\right) - Zc1\left(IR1 - IR2\right)\right] \\ &= j\sqrt{3}\left[\left(VR1 - VF1\right) - VR2 - Zc1\left(2IR1\right) + VF1\right] \end{aligned} \tag{33}$$

$$\frac{S1}{IR1} = j\sqrt{3}\left(2ZL1 - 1Zc1\right) \tag{34}$$

and for S2:

$$\begin{aligned} S2 &= \left(a^2 - a\right)\left(VR1 - VF1 + VF1\right) \\ &= j\sqrt{3}\left[\left(VR1 - VF1\right) + IR1\left(Zs1 + ZL1\right)\right] \end{aligned} \tag{35}$$

$$\frac{S2}{IR1} = j\sqrt{3}\left(2\ ZL1 - Zs1\right) \tag{36}$$

Using the results in (34) and (36), the resulting *a* and *b* vectors are:

$$a = Zc1 \tag{37}$$

$$b = -\frac{Zs1}{2} \tag{38}$$

The positive-sequence polarized phase distance unit has a definite reach, given by Vector *a*. Fig. 9 illustrates the threshold locus on the ZL1 plane.

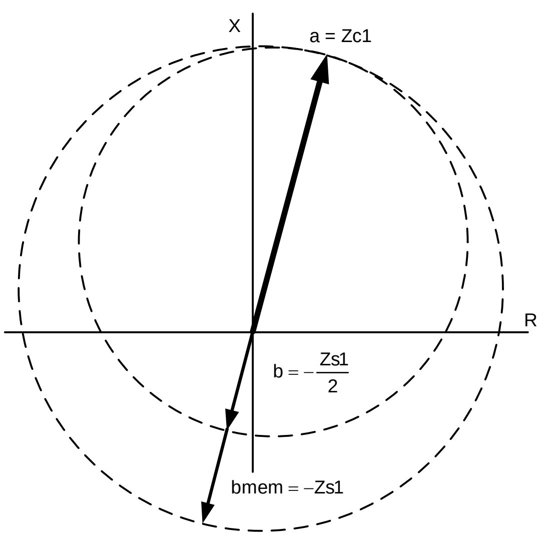

Fig. 9. Forward Fault Characteristic on the ZL1 Plane

D. Positive-Sequence Polarized BC Phase Distance Unit (Reverse Characteristics)

In this section, we will follow a similar procedure as described in Section III, Subsection B. Fig. 10 is the network used to derive the reverse fault characteristic for the BC phase distance element.

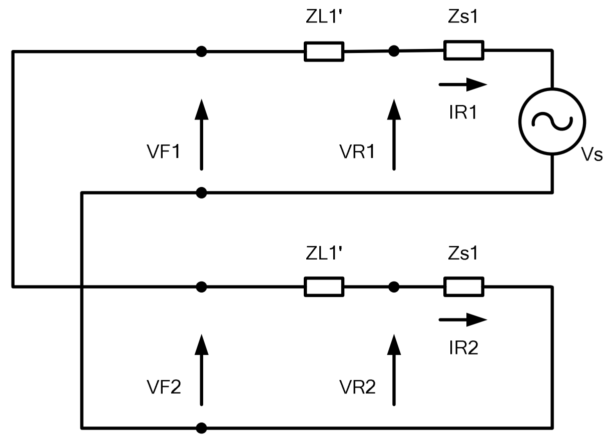

Fig. 10. Symmetrical Component Network for a Reverse BC Fault

Returning to (31), simplify to find a more useful expression for S1, based on the network shown in Fig. 10:

$$\begin{aligned} S1 &= (a^2 - a)\left[(VR1 - VR2) - Zc1\,(IR1 - IR2)\right] \\ &= j\sqrt{3}\left[\left(VR1 - VF1\right) - VR2 - Zc1\left(2IR1\right) + VF1\right] \\ &= j\sqrt{3}[\left(VR1 - VF1\right) - VR2 - Zc1\left(2IR1\right) \\ &\qquad + VR2 - ZL'\,IR1 \end{aligned} \tag{39}$$

$$\frac{S1}{(-IR1)} = j\sqrt{3}\left(2\ ZL1 + 2\ Zc1\right) \tag{40}$$

Returning to (32), simplify to find a more useful expression for S2:

$$\begin{aligned} S2 &= (a^2 - a)\left[(VR1 - VF1) + VF1\right] \\ &= j\sqrt{3}\left[\left(VR1 - VF1\right) - IR1\left(Zs1 + ZL1\right)\right] \end{aligned} \tag{41}$$

$$\frac{S2}{(-IR1)} = j\sqrt{3}\left(2\ ZL1 - Zs2\right) \tag{42}$$

Using the results in (40) and (42), the resulting *a* and *b* vectors are:

$$a = -Zc1 \tag{43}$$

$$b = -\frac{Zs1}{2} \tag{44}$$

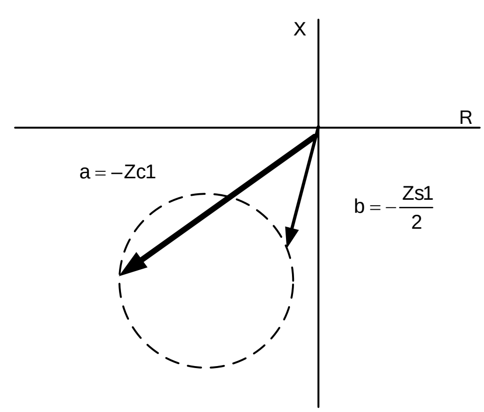

Fig. 11. Reverse Fault Characteristic on the ZL1′ Plane

E. Phase-to-Phase-to-Ground Faults

An interesting design choice is the detection of phase-to-phase-to-ground faults. These can be considered either as ground faults (due to the involvement of the ground return) or phase faults (due to their multiphase nature). The choice is a design choice of the manufacturer.

When single-pole trip, line distance protection schemes are designed, phase-to-phase-to-ground faults are to trip the three poles of the breaker. Only phase-to-ground faults, when detected, imply the opening of a single pole of the breaker. This argument, by itself, justifies detecting phase-to-phase-to ground faults with the phase elements. Other arguments include the operation of the phase selectors in the overall scheme.

The positive-sequence polarized phase distance unit described by (31) and (32) readily detects phase-to-phase-to-ground faults. Fig. 12 is the network used to derive the forward fault characteristic for the BCG phase distance element.

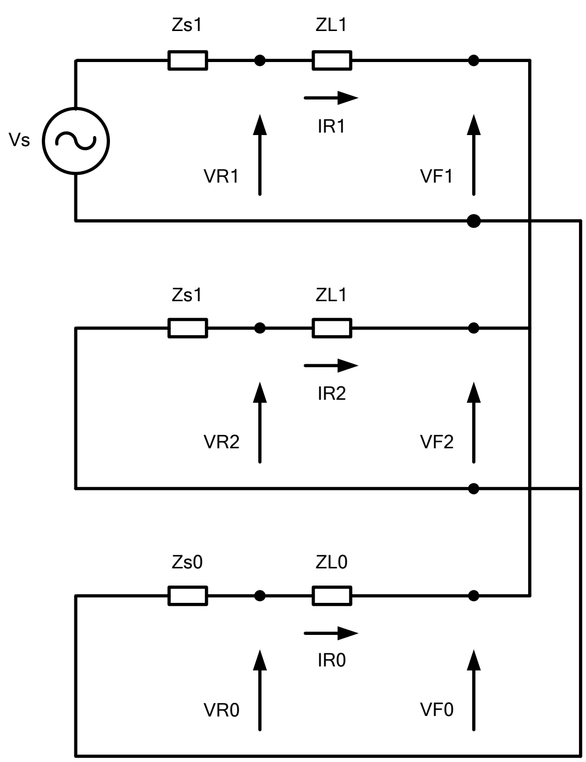

Fig. 12. Symmetrical Component Network for a Forward BCG Fault

Returning to (31), derive a more useful expression for S1 based on the network shown in Fig. 12:

$$\begin{aligned} S1 = j\sqrt{3}[&(VR1 - VF1) + VF1 - VF2) \\ &- Zc1\,(IR1 - IR2)] \end{aligned} \tag{45}$$

Using:

$$IR2 = -IR1\frac{Zs0 + ZL0}{Zs0 + ZL0 + Zs1 + ZL1} \tag{46}$$

(45) yields:

$$\frac{S1}{IR1} = j\sqrt{3}\left[\begin{aligned} &ZL1\left(1 + \frac{Zs0 + ZL0}{Zs0 + ZL0 + Zs1 + ZL1}\right) \\ &- Zc1\left(1 + \frac{Zs0 + ZL0}{Zs0 + ZL0 + Zs1 + ZL1}\right) \end{aligned}\right] \tag{47}$$

Returning to (32), derive a more useful expression for S2:

$$S2 = j\sqrt{3}[(VR1 - VF1) + VF1] \tag{48}$$

using (46):

$$\frac{S2}{IR1} = j\sqrt{3}\left[\begin{aligned} &ZL1\left(1 + \frac{Zs0 + ZL0}{Zs0 + ZL0 + Zs1 + ZL1}\right) \\ &+ Zs1\left(\frac{Zs0 + ZL0}{Zs0 + ZL0 + Zs1 + ZL1}\right) \end{aligned}\right] \tag{49}$$

Next, use (47) and (49) to find the *a* and *b* vectors:

$$a = Zc1 \tag{50}$$

$$\begin{aligned} b &= -Zs1\left[\frac{Zs0 + ZL0}{Zs1 + ZL1 + 2Zs0 + 2ZL0}\right] \\ &= f(ZL1) \end{aligned} \tag{51}$$

While Vector *a* is well-defined with a very definite and desired reach, Vector *b* is a function of ZL1 but in the negative direction. The result indicates that to find the shape requires an iterative process, because *b* is not in the same position for two different values of ZL1. However, what really matters is (50) because the element has a definite reach.

F. Three-Phase Faults and Memory in S2

For both the ground distance elements and the phase distance element, three-phase faults can be shown to present the same characteristic on the ZL1 plane. The *a* and *b* vectors can be shown to be the ones found for the self-polarized distance unit, described in (6) and (7).

The characteristic plotted in Fig. 3 shows that the circle boundary passes through at the distance element location. Neither the ground nor the phase element can detect the fault due to the lack of any voltage to calculate the positive-sequence polarizing quantity (S2). The distance unit should be able to detect these faults.

In general, memory action is provided, and S2 “remembers” the proper polarizing quantity after the fault when all voltages are zero. In most implementations of modern relays, S2 is implemented with memory for all phase and ground distance elements [6]. Besides the benefit associated with detecting three-phase faults right at the location of the distance unit, providing memory action enhances the ability of distance protection schemes to be applied in lines with series capacitors and increases fault detection ability.

For the positive-sequence polarized phase distance unit, a modified S2 polarizing vector is:

$$S2 = VB1mem - VC1mem \tag{52}$$

This translates to:

$$\begin{aligned} S2 &= j\sqrt{3}(Vs1) \\ &= j\sqrt{3}(VR1 + Zs1\ IR1) \end{aligned} \tag{53}$$

For a three-phase fault:

$$\frac{S2}{IR1} = j\sqrt{3}(ZL1 + Zs1) \tag{54}$$

Using (54) together with (37), the memory effect expands the circle with:

$$bmem = -Zs1 \tag{55}$$

In fact, it can be shown that polarizing the ground and phase distance elements with memory modifies Vector *b*, resulting in the following values.

For the ground distance unit:

$$bmem = -Zs1\left(\frac{2 + \frac{Zs0}{Zs1}}{2 + \frac{ZL0}{ZL1}}\right) \tag{56}$$

For the phase distance unit:

$$bmem = -Zs1 \tag{57}$$

These results are shown in Fig. 5 and Fig. 9. In the operation of these distance elements, the memory has a time constant, which varies with the design and purpose [6]. In traditional phase distance relay applications, the time constant should be enough to detect the zero-voltage, three-phase fault. Series capacitor applications require a longer time constant and associated logic that changes short and long memory constants.

When picturing the operation of these units, the *b* vector starts at *b* = bmem, and after a few time constants, its value approaches that of the "no-memory" mho circle.

G. Response to Other Fault Types and Phase Selector Criteria

Although not part of this paper, it has been shown elsewhere that all distance elements have a response to other types of faults [3]. However, with the methodology described previously, these characteristics can be derived.

Fig. 13 is an example of the composite response for an AG fault of all the ground distance elements. There are areas of the ZL1 plane where two or three of these elements will operate. The same is true for phase distance elements and their responses to other fault types.

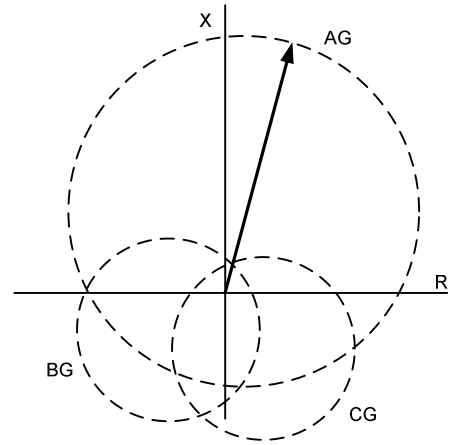

Fig. 13. Response of All Three Ground Distance Elements to an AG Fault on the ZL1 Plane

Interestingly, the responses of the elements do not overreach. Therefore, as a protective scheme design choice, the distance elements can detect all types of faults. The other choice could be to use a phase selector to enable only the type of distance unit appropriate for the type of fault. That way, only the AG element detects AG faults, since the fault detector has determined the type of fault.

H. Fault Resistance and Load Flow

Fault resistance and load flow are not considered when describing the characteristics of distance elements. The plots in Fig. 5 and Fig. 9 are provided and assume a radial system.

The appendix of this paper provides the complete derivations of the impedance plots on the apparent impedance plane, considering these two factors.

Fault resistance and load flow are more important in the analysis of the operation of distance units. During the testing of the characteristic's distance units, these two factors are generally not considered.

IV. Analysis of a Quadrilateral Ground Distance Unit

Quadrilateral distance units are provided in protection systems to cover ground faults. While their advantages over traditional mho ground distance units and ground overcurrent protection are a matter of discussion, these units are present in protective relays and merit consideration.

Implementing a quadrilateral element requires a reactance element, a resistance element, and a directional element.

A. Reactance Element

The reactance element for a ground directional element is defined by the following equations [2][3]:

$$S1 = VRA - Zc(IRA + kc0\ 3I0) \tag{58}$$

$$S2 = j\ IR2\ e^{jT} \tag{59}$$

The polarizing quantity S2 is a current; T is the "nonhomogeous correction angle" to be described later. It can be shown that I2 is the better choice when load is considered. However, I0 and (IA+K0 I0) are other possibilities for obtaining a reactance line.

Equation (58) is the same as (8) and can also be simplified to the expression in (16).

The polarizing quantity S2 is different. To simplify S2, a relatively nonacademic mathematical trick can be used.

$$\text{S2} = 0\,(\text{VR1} - \text{VF1}) + \text{j IR1}\, e^{jT} \tag{60}$$

$$\frac{\text{S2}}{\text{IR1}} = \frac{0}{0}\left(\text{ZL1} + \frac{\text{j } e^{jT}}{0}\right) \tag{61}$$

The end result is:

$$\frac{\text{S1}}{\text{S2}} = \frac{\text{ZL1} - \text{Zc1}\left(\dfrac{2 - \dfrac{\text{Zc0}}{\text{Zc1}}}{2 - \dfrac{\text{ZL0}}{\text{ZL1}}}\right)}{\text{ZL1} + \dfrac{\text{j } e^{jT}}{0\left(2 - \dfrac{\text{ZL0}}{\text{ZL1}}\right)}} \tag{62}$$

Equation (62) is not rigorously correct, but it allows for a criterion to obtain the expression for Vector *b*.

$$\text{b} = \infty\, e^{-j\left[90 - T + \text{ang}\left(2 + \frac{\text{Zc0}}{\text{Zc1}}\right)\right]} \tag{63}$$

Vector *a* is expressed in (19). Fig. 14 illustrates the graphical result. Vector *b* is a very large and infinite vector but with an angle. The reactance line is a circle with infinite radius.

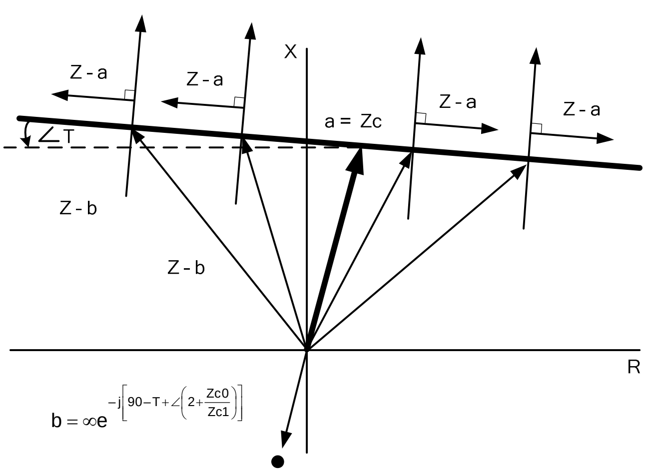

Fig. 14. Reactive Line Component of a Ground Quadrilateral Unit

B. Resistance Element

The resistive check for a quadrilateral element could be implemented with equations similar to (58) and (59), using S2 = IR2 and Zc = R [3]. Another approach is to actually calculate the resistive component of the fault [1][2].

Consider the symmetrical component representation of a ground fault with resistance, as shown in Fig. 15. The following is true:

$$\begin{aligned} \text{VF1} + \text{VF2} + \text{VF0} &= \text{If Rf} \\ \text{VRA} - \text{ZL1}(\text{IRA} + \text{kc0 3I0}) &= \text{If Rf} \end{aligned} \tag{64}$$

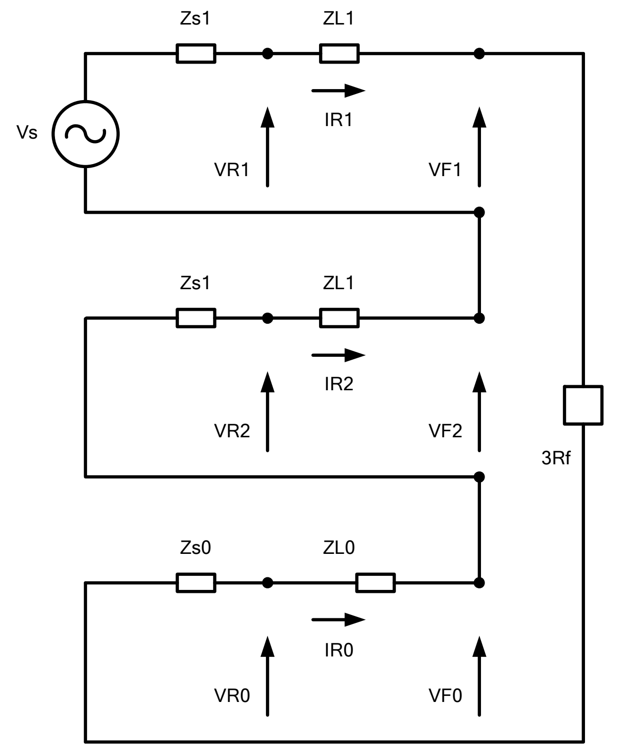

Fig. 15. Symmetrical Component Representation of an AG Fault With Fault Resistance (Rf)

If the protective relay is doing a DFT (discrete Fourier transform) or cosine filter calculation, the real and imaginary parts of the vectors are available. It follows that:

$$\begin{aligned} &\text{Im}\left\{\text{VRA}\left[\text{ZL1}(\text{IRA} + \text{kc0 3I0})^*\right]\right\} = \\ &\text{Im}\left\{(\text{If Rf})\left[\text{ZL1}(\text{IRA} + \text{kc0 3I0})^*\right]\right\} \end{aligned} \tag{65}$$

and:

$$\text{Rf} = \frac{\text{Im}\left\{\text{VRA}\left[\text{Zc1}(\text{IRA} + \text{kc0 3I0})^*\right]\right\}}{\text{Im}\left\{\left[\text{If } (\text{Zc1}(\text{IRA} + \text{kc0 3I0})^*\right]\right\}} \tag{66}$$

In one design choice [2], the If value was chosen to be:

$$\text{If} = 3\frac{(\text{IR0} + \text{IR2})}{2} \tag{67}$$

Equation (66) yields the fault resistance value (Rf), which for a radial system adds to the line impedance. This result is a straight calculation of Rf and cannot be plotted on an impedance plane. In fact, the directional element that keeps the reactance unit from detecting reverse faults cannot be plotted on the plane. The quadrilateral element is a combination of the reactance line, the resistive check, and a directional element.

V. Testing of Distance Elements

Relay engineers are curious. Most of them want to have a deep understanding of the devices that are connected to their power system. When the device fundamentals are understood, analysis and testing are simple. There are three types of testing that satisfy different functions of a numerical protective relay: acceptance, commissioning, and maintenance [7].

Acceptance testing involves the study of the protective device and its characteristics. Some users qualify protective devices over a period of time, studying and testing the characteristics provided by the device. This test is performed in a quiet environment, usually the company's protection laboratory, with ample time to verify the published characteristics. During this type of testing, the users have an opportunity to obtain an understanding of the relay principles.

Commissioning testing involves the actual installation of the protective device and the testing of the connections to it (e.g., polarities and phasing) and of the programmable logic. Some distance-unit reaches may be verified on the distance elements. The commissioning testing is by no means as involved as the acceptance testing, due to the timing and most likely the pressure to get the devices installed.

Maintenance testing involves a periodic and simple functional evaluation of the protective device. The testing takes advantage of the self-check algorithms in numerical protective relays and also the nature of the hardware design. Since the firmware is a software program, it is not expected that the features tested in the acceptance testing and the commissioning testing have changed. A visual inspection of the enable light, confirmation that the metering matches known levels, and testing of auxiliary contacts or devices should suffice. The maintenance testing interval is dependent on the data collected from the device. If the metering and oscillographic reports are accessed constantly, the maintenance interval may be longer than for a device that is totally isolated and not reporting any events.

It is hoped that the information in this section helps in understanding the actual operation of distance elements and acceptance testing. The implementation of the distance units has been tested and verified by the manufacturer and users several times, especially if the device is mature in the marketplace.

A. Apparent Impedance

The term apparent impedance is largely used in protective relaying literature when discussing distance elements. It is a calculation of the impedance in terms of the measured voltages and currents. For a simple radial system with no load flow, the apparent impedance can be readily used to verify the plots of Fig. 5 and Fig. 9 on the ZL1 plane. For load flow and fault resistance in a two-source network, the term is not a physical impedance. It is a measurement that implies apparent impedance, and it only represents an impedance (ZL1 in our discussion) if the system is radial. When fault resistance and load flow are considered, the apparent impedance is just a measurement.

1) Apparent Impedance for a Ground Fault

The network in Fig. 4 is also used to derive the expression for the apparent impedance. Therefore, for the AG fault in Fig. 4:

$$\begin{aligned} &\mathrm{VF1} + \mathrm{VF2} + \mathrm{VF0} = 0 \\ &(\mathrm{VR1} - \mathrm{ZL1\,IR1}) \\ &\quad + (\mathrm{VR2} - \mathrm{ZL1\,IR2}) + (\mathrm{VR0} - \mathrm{ZL0\,IR0}) = 0 \\ &\mathrm{VRA} + \mathrm{ZL1}\left[\mathrm{IRA} + \mathrm{kL0}\,(3\mathrm{I0})\right] = 0 \end{aligned} \tag{68}$$

Equation (68) simplifies to:

$$\mathrm{ZL1} = \frac{\mathrm{VRA}}{\mathrm{IRA} + \mathrm{kc0}\,(3\mathrm{I0})} \tag{69}$$

where kc0 is defined in (10) as a relay setting. It should be equal to KL0:

$$\mathrm{kL0} = \frac{\mathrm{ZL0} - \mathrm{ZL1}}{3\mathrm{ZL1}} \tag{70}$$

2) Apparent Impedance for a Phase-to-Phase Fault

The following is true for the network in Fig. 8:

$$\begin{aligned} &\mathrm{VF1} - \mathrm{VF2} = 0 \\ &(\mathrm{VR1} - \mathrm{ZL1\,IR1}) - (\mathrm{VR2} - \mathrm{ZL1\,IR2}) = 0 \\ &(\mathrm{VR1} - \mathrm{VR2}) - \mathrm{ZL1}(\mathrm{IR1} - \mathrm{IR2}) = 0 \end{aligned} \tag{71}$$

Equation (71) simplifies to:

$$\mathrm{ZL1} = \frac{\mathrm{VRB} - \mathrm{VRC}}{\mathrm{IRB} - \mathrm{IRA}} \tag{72}$$

B. Confirming the Distance Unit Characteristic

Symmetrical components are used to derive the distance element characteristics on the ZL1 plane. Assuming a radial system, this is how the impedance characteristics on the ZL1 plane are derived. The effect of load flow and fault resistance is neglected; their influence is easier to visualize on the apparent impedance plane when the operation of the unit is studied. Symmetrical component theory is fundamental for relay engineers. It makes three-phase power system analysis simple and understandable.

When testing a memory polarized distance unit, the way the prefault and faulted voltages are managed affects the test results. The memory effect will not be noticeable when the calculated fault voltages are maintained and the calculated fault current is increased until the unit trips. But if the prefault voltages are healthy and then fault quantities are applied, the memory effect will be noticeable.

Memory algorithms may vary among different manufacturers, but the decay of the polarizing memory voltage (S2) is, for most practical purposes, too fast to obtain the memory voltage characteristic. Moreover, the filtering of the voltages and currents may take some time, and the memory voltage may have already changed its value. However, by choosing an impedance point in between the no-memory and "with-memory" polarization, the behavior can be checked. This will be illustrated with the phase distance element.

1) Testing the Ground Distance Unit

For illustration, the parameters in Table I were chosen for the model.

TABLE I
GROUND DISTANCE-UNIT MODEL PARAMETERS

Parameter	Value	Description
Zc1	10 ∠80°	Positive-sequence reach
Zc0	30∠70°	Derived from the setting kc0 (10) that is normally a setting
kc0	0.674∠–14.93°	The actual kc0
Zs1	1∠85°	Positive-sequence source impedance
Zs0	3∠80°	Zero-sequence source impedance

Fig. 16 shows the corresponding mho circle derived from the parameters in Table I. Ten random points were chosen from the plot, and their impedances were calculated. For each point, the symmetrical component analysis yielded the corresponding test voltages and currents, as shown in Table II. By fixing the voltages and slowly increasing the current, the trip point is found when the distance unit trips. The fault magnitude listed in Table II corresponds, within the margins of the test equipment error, to the theoretical value, also shown in Table II.

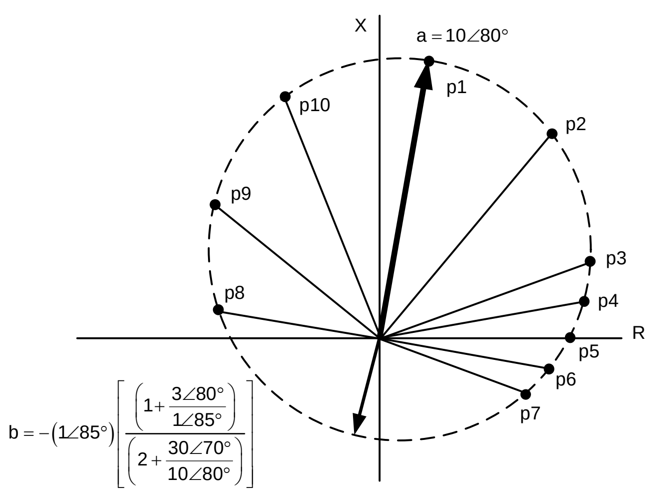

Fig. 16. Points on the Ground Distance Characteristic for the Example in Table I

TABLE II
TEST VOLTAGES AND CURRENTS

Point/ ZL1	Voltages and Currents	I
p1 10.00∠80°	Va = 63.68∠0°, Vb = 71.76∠–120.70°, Vc = 70.81∠122.78° Ia = **3.84**∠–74.15°, Ib = 0, Ic = 0	3.86
p2 8.95∠50°	Va = 64.24∠0°, Vb = 72.91∠–117.11°, Vc = 69.58∠126.03° Ia = **4.32**∠–44.31°, Ib = 0, Ic = 0	4.35
p3 6.04∠20°	Va = 65.36∠0°, Vb = 74.34∠–111.39°, Vc = 67.59∠131.34° Ia = **6.51**∠–14.07°, Ib = 0, Ic = 0	6.58
p4 4.87∠10°	Va = 66.15∠0°, Vb = 75.33∠–108.2°, Vc = 66.43∠134.44° Ia = **8.15**∠–14.07°, Ib = 0, Ic = 0	8.23
p5 3.81∠0°	Va = 67.41∠0°, Vb = 76.79∠–103.58°, Vc = 64.80∠138.85° Ia = **10.67**∠6.04°, Ib = 0, Ic = 0	10.73
p6 2.85∠–10°	Va = 69.69∠0°, Vb = 79.13∠–96.90°, Vc = 62.34∠145.61° Ia = **14.74**∠16.42°, Ib = 0, Ic = 0	14.87
p7 2.13∠–20°	Va = 73.67∠0°, Vb = 82.72∠–87.86°, Vc = 58.88∠155.35° Ia = **20.86**∠26.80°, Ib = 0, Ic = 0	21.07
p8 2.71∠170°	Va = 62.69∠0°, Vb = 66.17∠–146.27°, Vc = 79.30∠101.14° Ia = **13.92**∠–164.43°, Ib = 0, Ic = 0	13.96
p9 5.85∠140°	Va = 62.72∠0°, Vb = 69.42∠–130.35°, Vc = 74.06∠114.33° Ia = **6.45**∠–133.88°, Ib = 0, Ic = 0	6.48
p10 8.80∠110°	Va = 63.23∠0°, Vb = 70.83∠–124.37°, Vc = 72.05∠119.53° Ia = **4.32**∠–104.03°, Ib = 0, Ic = 0	4.36

This example illustrates the link between theoretical values and proper testing. The values in Table II come from the symmetrical component analysis of the fault.

The source impedance magnitude was not sufficient to make the memory effect noticeable; a larger source impedance should be chosen.

2) Testing the Phase Distance Element

To illustrate the phase distance-unit testing, the parameters from Table III were chosen.

TABLE III
PHASE DISTANCE-UNIT MODEL PARAMETERS

Parameter	Value	Description
Zc1	10∠80°	Positive-sequence reach
Zs1	10∠85°	Positive-sequence source impedance

In this example, the source impedance is larger and will allow testing of the memory effect on the distance characteristics.

Fig. 17 illustrates the plot of the phase distance element based on the parameters in Table III. The inner circle, as discussed in a previous section, is the characteristic with no-memory effect on the polarizing vector (S2).

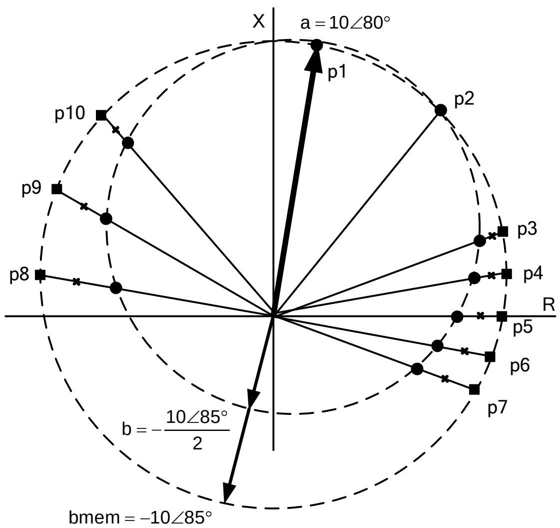

Fig. 17. Phase Distance-Unit Points on the Impedance Plane

Table IV shows the test voltages and currents used to confirm the no-memory characteristic. The test voltages are constant, while the current is increased until the phase distance element trips. The fault current magnitudes correspond to the expected values, within the margins of error of the test equipment and the device under test.

TABLE IV
TEST VOLTAGES AND CURRENTS WITH NO MEMORY

Point/ ZL1	Voltages and Currents	I
p1 10.00∠80°	Va = 70∠0°, Vb = 47.26∠–140.11°, Vc = 45.36∠138.07° Ia = 0, Ib = **3.03**∠–172.63°, Ic = **3.03**∠7.38°	3.05
p2 9.70∠50°	Va = 70∠0°, Vb = 53.57∠–146.19°, Vc = 39.22∠130.53° Ia = 0, Ib = **3.23**∠–157.84°, Ic = **3.23**∠22.16°	3.24
p3 8.65∠20°	Va = 70∠0°, Vb = 60.63∠–153.32°, Vc = 31.49∠120.18° Ia = 0, Ib = **3.85**∠–145.14°, Ic = **3.85**∠34.86°	3.87
p4 8.23∠10°	Va = 70∠0°, Vb = 63.12∠–155.97°, Vc = 28.51∠115.67° Ia = 0, Ib = **4.16**∠–142.04°, Ic = **4.16**∠37.97°	4.18
p5 7.74∠0°	Va = 70∠0°, Vb = 66.20∠–159.41°, Vc = 24.63∠109.02° Ia = 0, Ib = **4.60**∠–139.55°, Ic = **4.60**∠40.85°	4.61
p6 7.26∠–10°	Va = 70∠0°, Vb = 69.46∠–163.27°, Vc = 20.30∠99.86° Ia = 0, Ib = **5.14**∠–137.19°, Ic = **5.14**∠42.81°	5.17
p7 6.82∠–20°	Va = 70∠0°, Vb = 72.86∠–167.64°, Vc = 15.64∠85.70° Ia = 0, Ib = **5.77**∠–167.64°, Ic = **5.77**∠43.78°	5.23
p8 6.82∠170°	Va = 70∠0°, Vb = 21.87∠–113.73°, Vc = 64.39∠161.89° Ia = 0, Ib = **4.83**∠152.17°, Ic = **4.83**∠–27.84°	4.83
p9 7.74∠150°	Va = 70∠0°, Vb = 29.67∠–122.89°, Vc = 59.37∠155.19° Ia = 0, Ib = **4.04**∠157.12°, Ic = **4.04**∠–22.88°	3.88
p10 8.65∠130°	Va = 70∠0°, Vb = 35.73∠–129.06°, Vc = 54.99∠149.71° Ia = 0, Ib = **3.52**∠164.21°, Ic = **3.52**∠–15.79°	3.53

The outer characteristic is related to the memory effect on S2. It is impractical to test this characteristic. The memory algorithms in protective relays are constantly updated; the filtering behavior of the relays will make the memory value (S2) change almost instantly. Therefore, any values calculated with the outer-circle impedances will not yield any practical results.

Visually, note that the memory polarizing voltage is changing to the new faulted value and the circle starts from the outer boundary and decays finally to the inner, no-memory boundary.

To test this behavior, impedances in between the outer and inner circle can be chosen. The midpoint impedances shown in Fig. 17 are good choices. These values are the average of the two impedances at the same angle from the outer and inner circles.

Moreover, to test the memory behavior, apply the fault after healthy voltages have been present for a while. Both fault voltages and currents should correspond to the calculated values.

Table V describes the midpoint impedances and the fault voltages and currents used to verify the memory polarization of the phase distance element. Healthy prefault voltages and currents start the fault sequence.

TABLE V
TEST VOLTAGES AND CURRENTS WITH MEMORY

Point/ ZL1	Voltages and Currents
p3 9.51∠20°	Va = 70∠0°, Vb = 61.69∠–151.70°, Vc = 33.17∠118.19° Ia = 0, Ib = 3.69∠–143.373°, Ic = 3.69∠36.627°
p4 9.32∠10°	Va = 70∠0°, Vb = 64.71∠–153.75°, Vc = 31.01∠112.684° Ia = 0, Ib = 3.95∠–139.25°, Ic = 3.95∠40.75°
p5 9.08∠0°	Va = 70∠0°, Vb = 68.56∠–156.25°, Vc = 24.63∠104.70° Ia = 0, Ib = 4.31∠–134.94°, Ic = 4.31∠45.06°
p6 8.84∠–10°	Va = 70∠0°, Vb = 72.77∠–158.86°, Vc = 26.329∠94.64° Ia = 0, Ib = 4.76∠–131.29°, Ic = 4.76∠48.71°
p7 8.61∠–20°	Va = 70∠0°, Vb = 78.08∠–161.92°, Vc = 24.60∠80.10° Ia = 0, Ib = 5.35∠–127.85°, Ic = 5.35∠52.152°
p8 8.19∠170°	Va = 70∠0°, Vb = 25.95∠–107.19°, Vc = 67.08∠158.31° Ia = 0, Ib = 4.51∠147.66°, Ic = 4.51∠–32.34°
p9 8.66∠150°	Va = 70∠0°, Vb = 31.52∠–120.11°, Vc = 60.66∠153.29° Ia = 0, Ib = 3.85∠155.09°, Ic = 3.85∠–24.91°
p10 9.15∠130°	Va = 70∠0°, Vb = 36.45∠–127.95°, Vc = 55.59∠148.86° Ia = 0, Ib = 3.43∠163.46°, Ic = 3.43∠–16.54°

Fig. 18 illustrates the operation of the distance element and the fault sequence. Note the effects of the memory action. The memory voltage is decaying much slower than the actual positive-sequence voltage. As the polarizing voltage decays, there will be a point in the decay where there is not enough polarizing voltage to operate the distance unit. The element eventually drops out, visualized as the circle dropping from the full-memory characteristic to the steady-state characteristic, which is polarized by V1. The memory action, however, has allowed an expansion of the element.

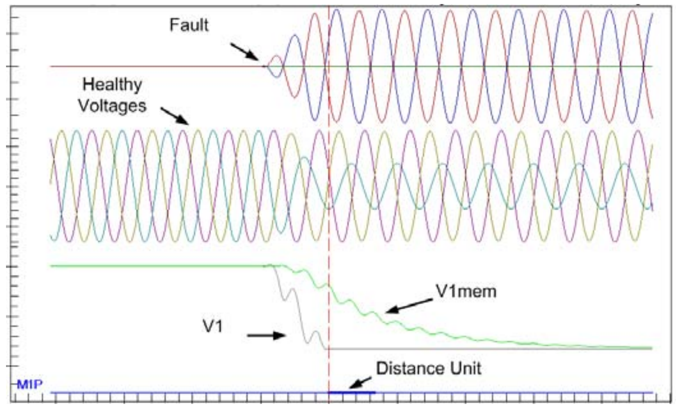

Fig. 18. Typical Fault Sequence Verifying the Voltages and Currents in Table V

3) Testing the Ground Reactance Distance Unit

To illustrate the testing of the ground reactance distance unit, the parameters in Table I will be used. The setting T compensates for any effect associated with the tilt on the reactance line.

The reactance setting T (58) will be set to:

$$T = \arg\left(2 + \frac{Zc0}{Zc1}\right) = -6.00 \tag{73}$$

This value makes the reactance line horizontal, as seen in Fig. 19.

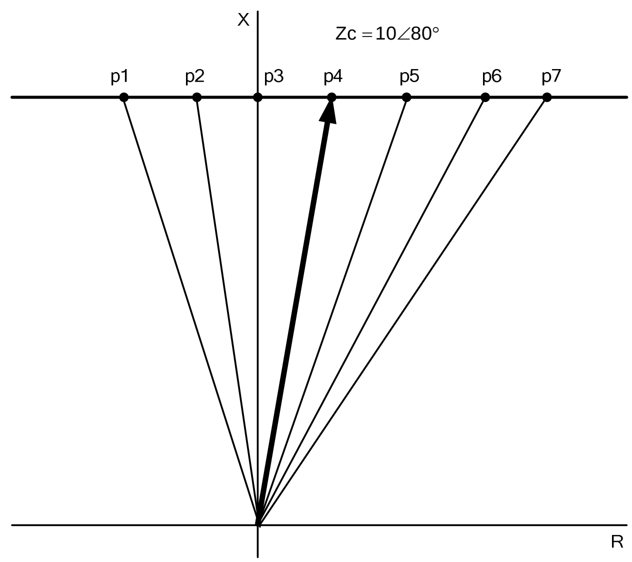

Fig. 19. Points on the Reactance Ground Distance Characteristic

Table VI represents seven points on the reactance line. These impedances lie on the threshold of operation. The recorded current in the right-hand column is within the margin of error in the test equipment and the protective relay.

TABLE VI
TEST VOLTAGES AND CURRENTS FOR FIG. 19

Point/ ZL1	Voltages and Currents	I
p1 10.38∠108.33°	Va = 64.15∠0°, Vb = 70.34∠–140.11°, Vc = 7.200∠138.07° Ia = **3.72**∠–102.33°, Ib = 0, Ic = 0	3.75
p2 9.98∠99.21°	Va = 63.71∠0°, Vb = 70.72∠–123.03°, Vc = 71.83∠120.44° Ia = **3.85**∠–93.21°, Ib = 0, Ic = 0	3.87
p3 8.65∠20°	Va = 63.53∠0°, Vb = 71.10∠–122.06°, Vc = 71.56∠121.54° Ia = **3.89**∠–83.59°, Ib = 0, Ic = 0	3.91
p4 10∠80°	Va = 63.67∠0°, Vb = 71.42∠–121.00°, Vc = 71.21∠122.56° Ia = **3.83**∠–74.00°, Ib = 0, Ic = 0	3.86
p5 10.42∠70.94°	Va = 64.08∠0°, Vb = 71.65∠–120.01°, Vc = 70.84∠123.38° Ia = **3.70**∠–64.93°, Ib = 0, Ic = 0	3.73
p6 11.076∠62.76°	Va = 64.66∠0°, Vb = 71.78∠–119.19°, Vc = 70.50∠123.92° Ia = **3.51**∠–56.76°, Ib = 0, Ic = 0	3.54
p7 6.82∠–20°	Va = 65.30∠0°, Vb = 71.83∠–118.59°, Vc = 70.21∠124.20° Ia = **3.29**∠–49.62°, Ib = 0, Ic = 0	3.32

C. Comments on the Impedance Testing Results

The results obtained, the description of the testing, and the theory described in the previous sections allow for discussing a few interesting issues.

1) The Plot Is on the ZL1 Plane

The impedance plots correspond to the ZL1 plane. The characteristics represent actual impedances in a radial network for which the distance unit will operate. For purposes of testing the unit's characteristic, a radial network is assumed.

Plots that correspond to other planes should not be confused or used together with the distance element characteristic plots. For example, the impedance-based ground directional element plots its characteristics on a V2/I2 or V0/I0 plane.

2) The Plot Is for Faults in the Forward Direction

Fig. 7 and Fig. 11 illustrate the characteristics of distance units for faults in the reverse direction. They represent the impedance values on the reverse impedance plane (ZL1′) for which the element will operate and are a totally different plane than the forward fault impedance plane (ZL1) shown in Fig. 5 and Fig. 9.

Reverse fault characteristics are not possible to test in practice because distance elements are generally supervised by directional elements that keep distance units operating in the forward direction only.

3) Use Symmetrical Components When Testing

The distance element theory is based on symmetrical components. More than any other aspect of power systems, symmetrical components are the foundation of power system protective relaying and, in particular, distance elements. The faulted voltages and currents presented earlier were obtained from symmetrical component analysis of the faults.

A traditional way to estimate faulted voltages and currents for an AG ground fault, for example, is by using a phasor diagram, as illustrated in Fig. 20. The faulted voltage is decreased, and the faulted current is modified according to the impedance magnitude and angle requirement. While this methodology provides a "quick and dirty" way to make an estimate, it is not based on any theory. The unfaulted voltages remain with their healthy values, and the angles between voltages remain constant. This is not what happens in a real power system.

Moreover, as illustrated in Fig. 20, increasing the faulted current angle results in the faulted current and the faulted voltage being in phase. This condition will not occur in a transmission network. Directional elements will not declare the fault to be in the forward direction because, for ground faults, it is assumed that the source is reactive and not resistive, as equal angles will suggest.

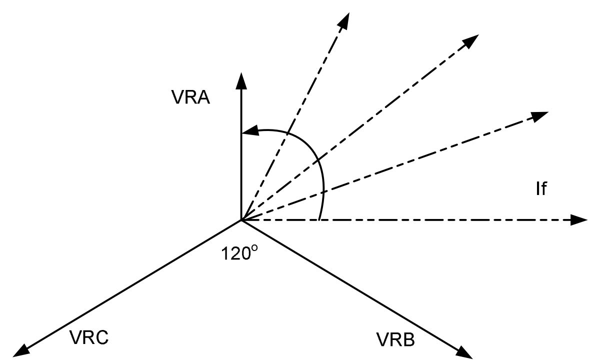

Fig. 20. Fixing Voltages and Changing the Fault Current Does Not Accurately Represent Faults in a Power System

4) Directional Elements Supervise Distance Units

If symmetrical components are used to calculate faulted voltages and currents, the directional units supervising the distance elements will be transparent to the testing procedure.

Directional unit theory is based on symmetrical components as well. For example, for ground directional elements, the source impedance determines that the direction to the fault is in the forward direction (the measurement is –Zs for a forward fault).

The use of symmetrical components and a known source impedance ensures that directional elements will not block the operation of distance units.

VI. Apparent Impedance, Load Flow, and Fault Resistance

The theory and testing covered in the previous sections illustrate a radial network and the graphical view of all the impedances that would make the distance unit operate. Load flow and fault resistance are factors that are present in all line distance protection schemes.

The ZL1 plane is not the proper plane to visualize the operation of the distance units when there is load flow and fault resistance. The fault resistance is not part of any of the three symmetrical component networks (positive, negative, and zero sequence).

The proper plane to use is the apparent impedance plane. For ground faults, the plane is defined by (67). For phase faults, the plane is defined by (70). In the appendix of this paper, the impedance circles are derived for both the ground and phase distance elements.

It is mathematically very difficult to find impedance locus on any plane to describe the distance element characteristics because the distribution of currents is different depending on the faulted impedance and load flow. With the use of the equations derived in the appendix, it is simpler to visualize the location of the apparent impedance measured with respect to the distance-unit characteristics.

Another important point is that all the information is available from the local measurements to calculate the apparent impedance.

To illustrate the plotting on the apparent impedance plane, the system in Fig. 21 is used. The end characteristic on the left-hand side is plotted, and a large source impedance is selected to illustrate the mho circle behavior compared to the radial characteristics discussed before.

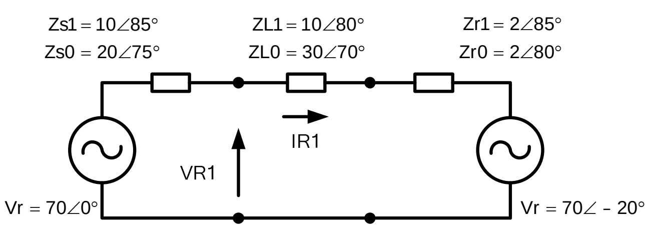

Fig. 21. Sample Power System Used to Illustrate the Impedance Plot With Rf and Load Flow

A. Ground Distance Unit

Using the impedances in Fig. 21 and the network shown in Fig. 22, circuit analysis techniques can solve for the network. The distance-unit settings are the same as the ones shown in Table I.

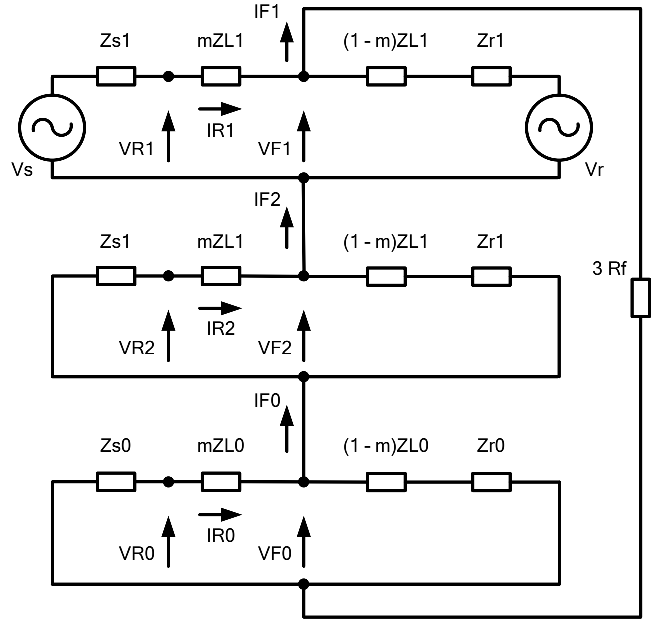

Fig. 22. Two-Source Symmetrical Component Representation of a Ground Fault

Solving for m = 1 and Rf = 0, only the load flow effect is illustrated in Fig. 23. The figure shows a computer-generated graph based on the equations derived in the appendix. As reported in other literature, mho circles adjust appropriately to prevent overreach due to the direction of the load flow [3]. The pivot point is the unit's reach.

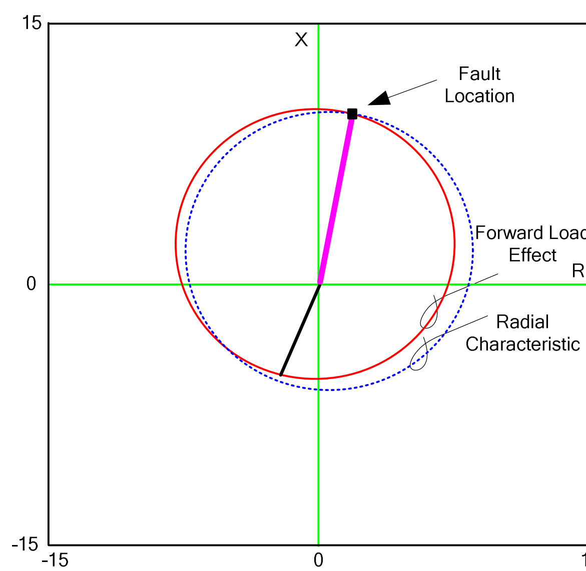

Fig. 23. Load Flow Effect on the Apparent Impedance Characteristic

For the same point (m = 1) and the same load flow but with fault resistance (Rf = 0.5 ohms), Fig. 24 illustrates the plot of the apparent impedance and the distance-unit characteristics. In the apparent impedance plane, the Rf component is not a straight line, and moreover, the apparent Rf component is magnified several times due to the strong remote source.

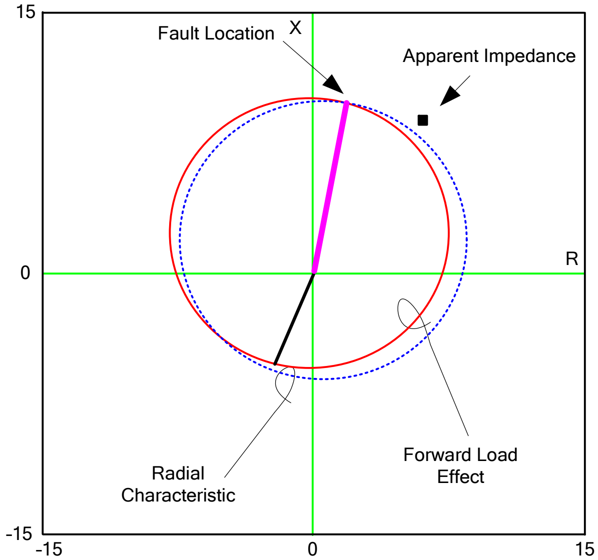

Fig. 24. Load Flow and Fault Resistance Apparent Impedance

The advantage of using a computer program to calculate the voltages and currents of the network is that a point on the plot can be found. With this point, the voltages and currents can be calculated.

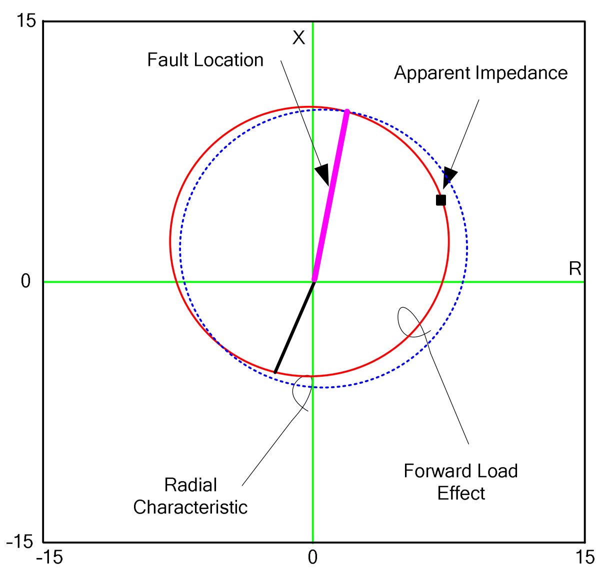

Fig. 25. Boundary Fault at m = 0.5 and Rf = 4.0

TABLE VII
TEST VOLTAGES AND CURRENTS FOR APPARENT IMPEDANCES IN FIG. 23, FIG. 24, AND FIG. 25

Point/ ZApp	Voltages and Currents	I
p1 10.00∠80°	Va = 37.33∠0°, Vb = 67.94∠–124.33°, Vc = 69.02∠113.01° Ia = **2.88**∠–71.83°, Ib = 0.53∠–143.88°, Ic = 1.67∠106.12°	2.89
p2 10.47∠53.82°	Va = 39.57∠0°, Vb = 67.77∠–112.93°, Vc = 69.51∠124.88° Ia = **2.97**∠–46.46°, Ib = 0.69∠–137.72°, Ic = 1.67∠123.59°	No trip
p3 8.63∠30.02°	Va = 40.94∠0°, Vb = 75.50∠–105.75°, Vc = 72.96∠146.88° Ia = **3.19**∠–18.63°, Ib = 1.12∠–93.09°, Ic = 1.08∠145.79°	3.20

Each point on the apparent impedance plane can be plotted using the expressions in the appendix; examples are shown in Fig. 23, Fig. 24, and Fig. 25. Distance units are monitoring the power system constantly. When there is a fault, the numerical algorithm calculates the phasor values of the voltages and currents. It therefore follows that the impedance trajectory on the apparent impedance plane can be plotted over time.

Numerical distance units make calculations in discrete time periods called processing intervals. It is therefore possible to evaluate the impedance trajectory to the fault over time. In Fig. 26, a plot of the impedance trajectory is shown. Note that at each processing interval, the apparent impedance changes. The apparently random motion until the impedance stabilizes to the actual fault apparent impedance is also due to the filtering algorithm used.

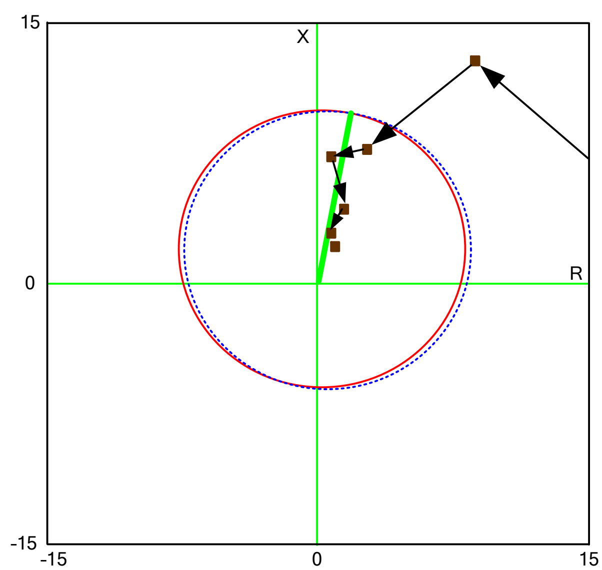

Fig. 26. Impedance Trajectory for a Ground Fault at m = 0.25 and Rf = 0.5

B. Phase Distance Unit

The symmetrical component network interconnection for a phase-to-phase fault is shown in Fig. 27. The system parameters are the ones shown in Fig. 21. The appropriate *a* and *b* vectors for the fault resistance and load flow are derived in the appendix. This unit also exhibits a beneficial tilt due to load flow.

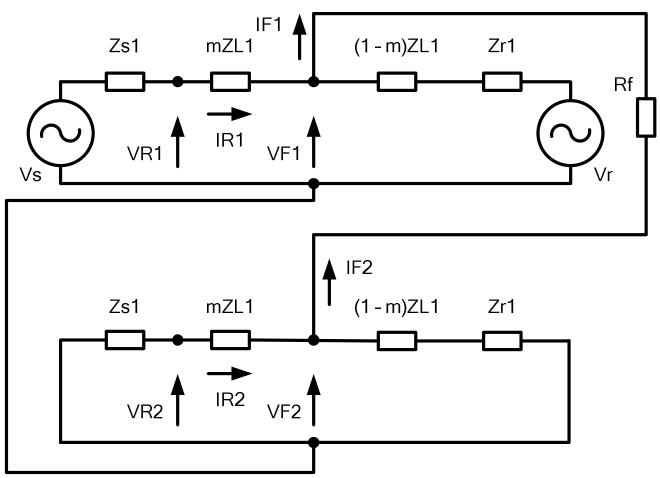

Fig. 27. Two-Source Symmetrical Component Representation of a Phase-to-Phase Fault

Fig. 28 illustrates the impedance trajectory for a BC fault. The expressions (85) and (86), derived in the appendix, accurately describe the apparent impedance trajectory.

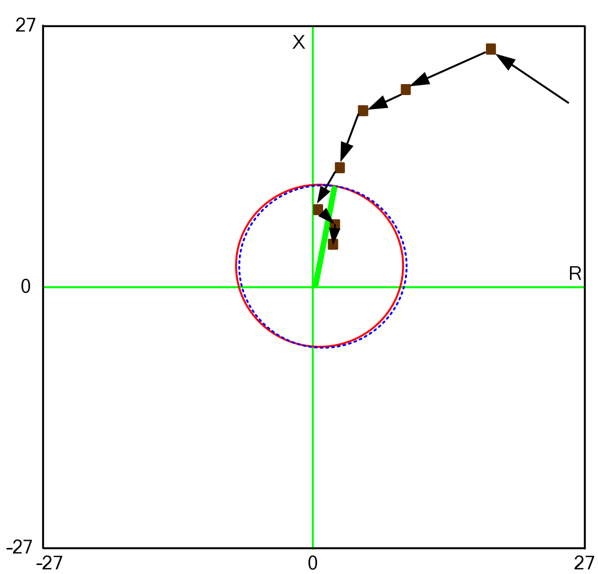

Fig. 28. Impedance Trajectory for a Phase-to-Phase Fault at m = 0.5 and Rf = 0.5

C. Ground Reactance Element

The load and fault resistance characteristic equations are derived in the appendix. The Vector *b* equation (90) determines the angle at which the reactance line tilts. The ground reactance line defined by (57) and (58) is an adaptive reactance line because it adjusts its tilt depending on the load flow. Quadrilateral distance units tend to overreach due to load flow and fault resistance. The ground reactance element pivots on the unit's reach (Zc) with a negative angle for loads in the forward direction (export) and a positive angle for loads in the reverse direction (import), as illustrated in Fig. 29.

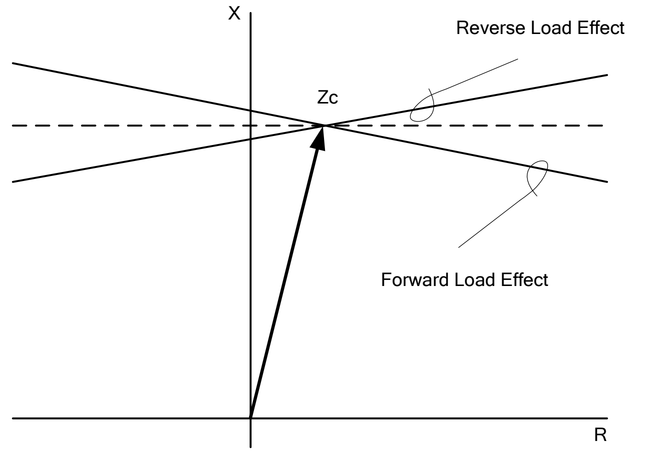

Fig. 29. Adaptive Ground Reactance Unit Characteristic

Even with the adaptive characteristics described by the equations derived in the appendix, there is a chance of overreaching. The setting T provides perfect adjustment due to load flow and fault resistance and will adjust for the nonhomogeneity of the negative-sequence network impedances. The angle T should be the difference between negative-sequence current at the fault (IF2) with respect to the negative-sequence current measured at the distance-unit location (used to polarize the reactance line) [7].

At the element's reach (usually Zone 1 is the most critical), the expression for T is:

$$T = \arg\left(\frac{Zs1 + ZL1 + Zr1}{Zr1}\right) \tag{74}$$

The network in Fig. 21 illustrates the adaptability of this element. The solution of the currents and voltages is obtained by solving the network in Fig. 22. The fault will be located at the end of the line (the reach of the element is also set to this length), and a few Rf values will be used. For the network in Fig. 21, T = –2.273.

Fig. 30 illustrates selected points used to derive the test voltages and currents in Table VIII.

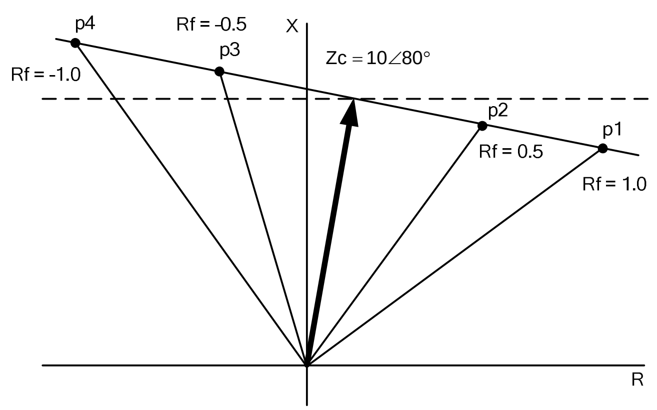

Fig. 30. Test Points on the Reactance Line With Different Fault Resistance Values

TABLE VIII
TEST VOLTAGES AND CURRENTS FOR APPARENT IMPEDANCES IN FIG. 30

Point/ ZApp	Voltages and Currents	I
p1 12.38∠35.97°	Va = 44.94∠0°, Vb = 67.81∠–17.44°, Vc = 69.83∠130.92° Ia = **2.91**∠–29.26°, Ib = 0.83∠–130.45°, Ic = 1.65∠134.07°	2.91
p2 10.47∠53.82°	Va = 39.57∠0°, Vb = 67.77∠–112.93°, Vc = 69.51∠124.88° Ia = **2.97**∠–46.46°, Ib = 0.69∠–137.72°, Ic = 1.69∠123.59°	2.99
p3 12.03∠106.78°	Va = 41.16∠0°, Vb = 68.35∠–135.53°, Vc = 68.53∠101.68° Ia = **2.57**∠–97.61°, Ib = 0.45∠–136.83°, Ic = 1.57∠89.26°	2.61
p4 16.63∠125.50°	Va = 48.42∠0°, Vb = 68.83∠–140.44°, Vc = 68.21∠97.40° Ia = **2.13**∠–115.07°, Ib = 0.51∠–123.08°, Ic = 1.44∠81.23°	2.12

VII. FINAL DISCUSSION

This paper presented a general technique to derive distance elements, as well as deriving the characteristics of three distance units. Knowing the distance element theory allows for the understanding of the units, which in turn facilitates the acceptance testing sequence.

The distance-unit characteristics are plotted on an impedance plane. The positive-sequence impedance plane represents the behavior of the units, and a radial system with no fault resistance or load flow is assumed.

The characteristics imply faults in the forward direction. Distance units also have characteristic plots for the reverse direction.

When considering fault resistance and load flow, the apparent impedance plane describes the measurement with respect to the impedance characteristic.

Mho distance elements are directional with a desirable response in the forward and reverse fault impedance characteristics. Memory action on these units allows for an expansion larger than the normal expansion due to the source impedance.

When the fault voltages are fixed and the fault current is increased until the distance element trips, testing will show the steady-state (no-memory) characteristics. Using two states during the testing, with the prefault state considering only healthy voltages, the influence of the memory effect in distance units can be observed when the fault state is applied.

Quadrilateral distance units are composed of a reactance unit, a resistance check unit, and a directional element. The reactance unit presented is a circle of infinite radius. The directional element and the resistance check of a particular design cannot be plotted on the apparent impedance plane.

The impedance trajectories can be plotted on the apparent impedance planes for both the ground and phase distance units. These plots represent a measurement and not an actual impedance.

VIII. APPENDIX

This section introduces how to find the distance element characteristics on the apparent impedance plane. On this plane, the graphical characteristics of the distance elements can be observed, along with the influence of load and fault resistance. Only for radial networks, the ZL1 plane is equal to the apparent impedance plane.

A. Positive-Sequence Polarized Ground Distance Unit (Apparent Impedance Plane Forward Characteristics)

Fig. 31 illustrates a simple two-source network and a Phase A-to-ground fault. The ground distance unit is characterized by (8) and (9).

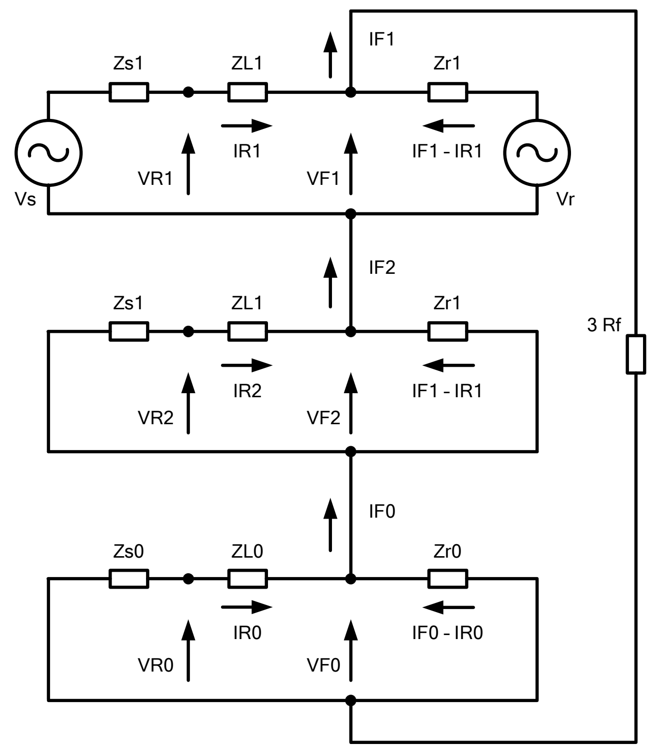

Fig. 31. AG Fault Symmetrical Component Network Interconnection

Returning to (8), simplify to find a more useful expression for S1:

$$S1 = VRA - Zc\left[IRA + kc0\,(3IR0)\right] \quad (75)$$

Equation (75) can be simplified to:

$$\frac{S1}{\left[IA + kc0\,(3I0)\right]} = \frac{VA}{\left[IA + kc0\,(3I0)\right]} - Zc1 \quad (76)$$

Returning to (9), simplify to find a more useful expression for S2:

$$\begin{aligned} S2 &= VR1 + VR2 + VR0 - VR2 - VR0 \\ &= VRA - VR2 - VR0 \\ &= VRA + Zs1\,IR2 + Zs0\,IR0 \end{aligned} \tag{77}$$

$$\frac{S2}{IA + kc0\,(3I0)} = \frac{VRa}{IA + kc0\,(3I0)} + Zs1\left(\frac{\frac{IR2}{IR1} + \frac{IR0}{IR1}\frac{Zs0}{Zs1}}{1 + \frac{IR2}{IR1} + \frac{Zc0}{Zc1}\frac{Zs0}{Zs1}}\right) \tag{78}$$

The results in (76) and (78) can be used in (2) to find the *a* and *b* vectors:

$$a = Zc1 \tag{79}$$

$$b = -Zs1\left(\frac{\frac{IR2}{IR1} + \frac{IR0}{IR1}\frac{Zs0}{Zs1}}{1 + \frac{IR2}{IR1} + \frac{Zc0}{Zc1}\frac{Zs0}{Zs1}}\right) \tag{80}$$

B. Positive-Sequence Polarized Phase Distance Unit (Apparent Impedance Plane Forward Characteristics)

Fig. 32 is the symmetrical component network interconnection of a two-source power system for a BC fault used to analyze the positive-sequence polarized phase distance unit defined by (31) and (32).

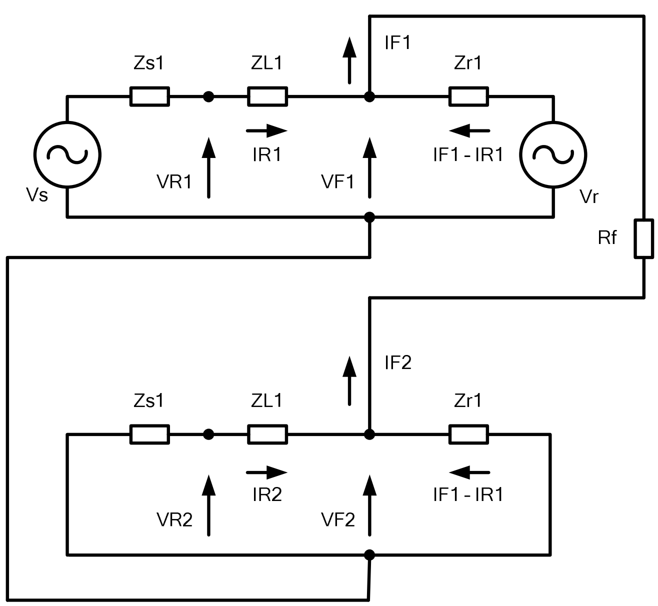

Fig. 32. BC Fault Symmetrical Component Network Interconnection

Returning to (31), find a more useful expression for S1:

$$S1 = (VRB - VRC) - Zc\,(IRB - IRC) \tag{81}$$

Equation (81) yields:

$$\frac{S1}{(IRB - IRC)} = \frac{(VRB - VRC)}{(IRB - IRC)} - Zc \tag{82}$$

Returning to (32), find a more useful expression for S2:

$$\begin{aligned} S2 &= VB1 - VC1 + VB2 - VB2 - VC2 + VC2 \\ &= VB - VC - VB2 + VC2 \end{aligned} \tag{83}$$

$$\frac{S2}{IRB - IRC} = \frac{VB - VC}{IRB - IRC} - Zs1\left(\frac{1}{1 - \frac{IA1}{IA2}}\right) \tag{84}$$

Using (82) and (84), the resulting *a* and *b* vectors are:

$$a = Zc1 \tag{85}$$

$$b = -Zs1\left(\frac{1}{1 - \frac{IA1}{IA2}}\right) \tag{86}$$

C. Ground Reactance Distance Unit (Apparent Impedance Plane Forward Characteristics)

The reactance element is defined by (57) and (58). Finding the S1 and S2 vectors on the apparent impedance plane follows:

$$\frac{S1}{\left[IA + kc0\,(3I0)\right]} = \frac{VA}{\left[IA + kc0\,(3I0)\right]} - Zc1 \tag{87}$$

$$S2 = 0\,VA + j\,IR2\,e^{jT} \tag{88}$$

With S2, a rigorous mathematical procedure is not practical, as shown in (89).

$$\frac{S2}{\left[IA + kc0\,(3I0)\right]} = \frac{0}{0}\left(\frac{VA}{\left[IA + kc0\,(3I0)\right]} + j\,\frac{IR2\,e^{jT}}{0\left[IA + kc0\,(3I0)\right]}\right) \tag{89}$$

The division by zero is infinity, and what matters is the angle.

$$b = \infty\,e^{-j\left[90 - T + ang\left(1 + \frac{IA1}{IA2} + \frac{IA0}{IA2}\frac{Zc0}{Zc1}\right)\right]} \tag{90}$$

IX. References

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[7] SEL-421 Reference Manual. Available: http://www.selinc.com/instruction_manual.htm.

X. Biography

Fernando Calero received his BSEE in 1986 from the University of Kansas, his MSEE in 1987 from the University of Illinois (Urbana-Champaign), and his MSEPE in 1989 from the Rensselaer Polytechnic Institute. From 1990 to 1996, he worked in Coral Springs, Florida, for the ABB relay division in the support, training, testing, and design of protective relays. Between 1997 and 2000, he worked for Itec Engineering, Florida Power and Light, and Siemens. Since 2000, Mr. Calero has been an application engineer in international sales and marketing for Schweitzer Engineering Laboratories, Inc., providing training and technical assistance.

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