Z = V/I DOES NOT MAKE A DISTANCE RELAY

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INTRODUCTION

Distance relays can provide effective transmission line protection. Their characteristics have usually been created from comparators and various combinations of voltages and currents.

The mho characteristic, for example, is a popular design because it can be made from a single comparator, has well-defined reach, is inherently directional, and can be made to tolerate fault resistance quite well without serious overreaching errors due to load.

Quadrilateral characteristics traditionally require four comparators; e.g., one for each side of the characteristic.

Because of the variety of fault types possible on a three-phase circuit, distance relays must be available to respond to the voltages and currents associated with six different fault loops.

The number of relay elements required for complete schemes is usually quite large. For example, a four-zone phase and ground mho distance relay requires 24 comparators. Quadrilateral characteristics require even more.

One approach to making a distance relay with a computer is to calculate the apparent impedance Z = V/I, and then check if that impedance is inside some geometric shape, like a circle or box. The hope is that one impedance calculation (per loop) might be useful for all zones.

Although relays have been made using this approach, their performance suffers under many practical conditions of load flow and fault resistance.

This paper examines the Z = V/I approach, and shows the degradations due to load flow and fault resistance. It shows us that calculating Z = V/I and testing Z against a circle passing through the origin is equivalent to a self-polarized mho element -- generally a poor performer for distance protection.

This paper shows some much better methods used in numerical relays, and emphasizes that these methods have their roots in better polarization methods.

Z = V/I AS A DISTANCE MEASUREMENT

Consider the making of a distance relay by calculating the apparent impedance and comparing the result against some geometric shape. In this paper, we refer to this approach as the Z = V/I method. This method is appealing in that it only requires one impedance calculation per fault loop. Multiple zones (or geometric shapes) only require more geometric tests of the result. These apparent impedance equations are listed in Table 1.

Table 1: Z = V/I Apparent Impedance Equations

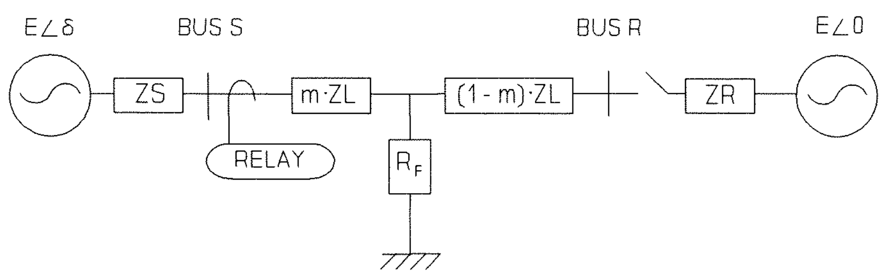
Fault Loop	Equations
AG	$V_A/[I_A + k0 \cdot I_R]$
BG	$V_B/[I_B + k0 \cdot I_R]$
CG	$V_C/[I_C + k0 \cdot I_R]$
AB	$[V_A - V_B]/[I_A - I_B]$
BC	$[V_B - V_C]/[I_B - I_C]$
CA	$[V_C - V_A]/[I_C - I_A]$

First look at the AG element performance for an AG fault on the radially configured system in Figure 1. The Aϕ voltage at Bus S is:

$$V = m \cdot Z_{1L} \cdot (I_A + k0 \cdot I_R) + R_F \cdot I_F \qquad \text{Equation 1}$$

where:

V = Aϕ voltage measured at Bus S
m = per-unit distance to the fault from Bus S
Z_{1L} = positive-sequence line impedance
I_A = Aϕ current measured at Bus S
$k0 = (Z_{0L} - Z_{1L})/(3 \cdot Z_{1L})$ (Z_{0L} = zero-sequence line impedance)
I_R = residual current measured at Bus S
R_F = fault resistance
I_F = total current flowing through R_F



$Z_{1S} = Z_{1L} \cdot (0.1)$
$Z_{0S} = 3 \cdot Z_{1S}$
PTR = 3500:1
CTR = 320:1
m = 0.85
kV = 400

$Z_{1L} = 10 + j \cdot 110\ \Omega$ pri.
$= 0.91 + j \cdot 10.06\ \Omega$ sec.
$Z_{0L} = 3 \cdot Z_{1L}$

$Z_{1R} = Z_{1S}$
$Z_{0R} = Z_{0S}$

Figure 1: System Single Line Diagram

Convert Equation 1 into an impedance measurement by dividing every term by I, where $I = (I_A + k0 \cdot I_R)$. This yields:

$$Z = \frac{V}{I} = m \cdot Z_{1L} + R_F \cdot \frac{I_F}{I} \qquad \text{Equation 2}$$

Z includes the line impedance to the fault plus $R_F \cdot (I_F/I)$. For the radial system, $\angle I_F = \angle I$ and Z accurately measures the reactance to the fault. Figure 2 shows the resistance and reactance impedance measured by the relay for an AG fault at m = 0.85, with $R_F = 4.6\Omega$ secondary (or 50Ω primary given PTR/CTR = 3500/320). Because $R_F \cdot (I_F/I)$ is all real, $\mathrm{Im}(V/I) = m \cdot |X_{1L}|$, regardless the magnitude of R_F.

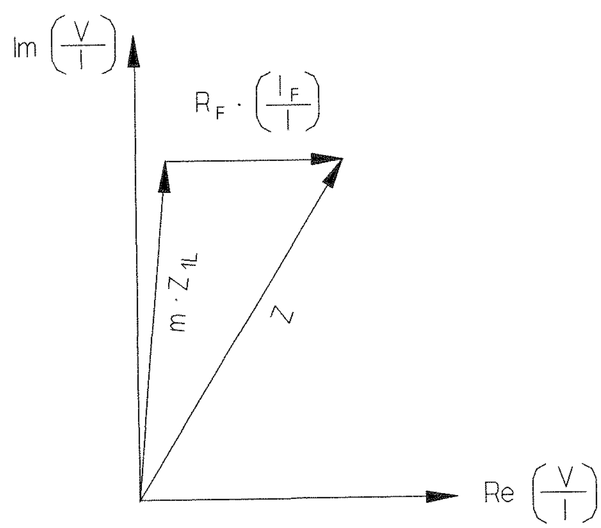


Figure 2: AG Apparent Impedance Method Correctly Measures Reactance to Fault for a Radial Line

This suggests that we can define a zone of ground distance protection with two reactance and resistance thresholds; i.e., the geometric test is a rectangle.

Figure 3 shows how these threshold checks enclose AG faults up to m = 0.85 with R_F less than 9.2Ω secondary. Reactance Threshold 2 and Resistance Threshold 2 define the desired reactance and resistance reach thresholds respectively. Reactance Threshold 1 and Resistance Threshold 1 restrict the zone definition to mostly the first quadrant in the impedance plane. Their small negative settings accommodate slight measurement errors near either axis (Im[V/I] or Re[V/I]). These later thresholds must be replaced with a separate directional element to insure directional security.

Thus, the first major problem to note is the Z = V/I approach is not inherently directional for ground faults.

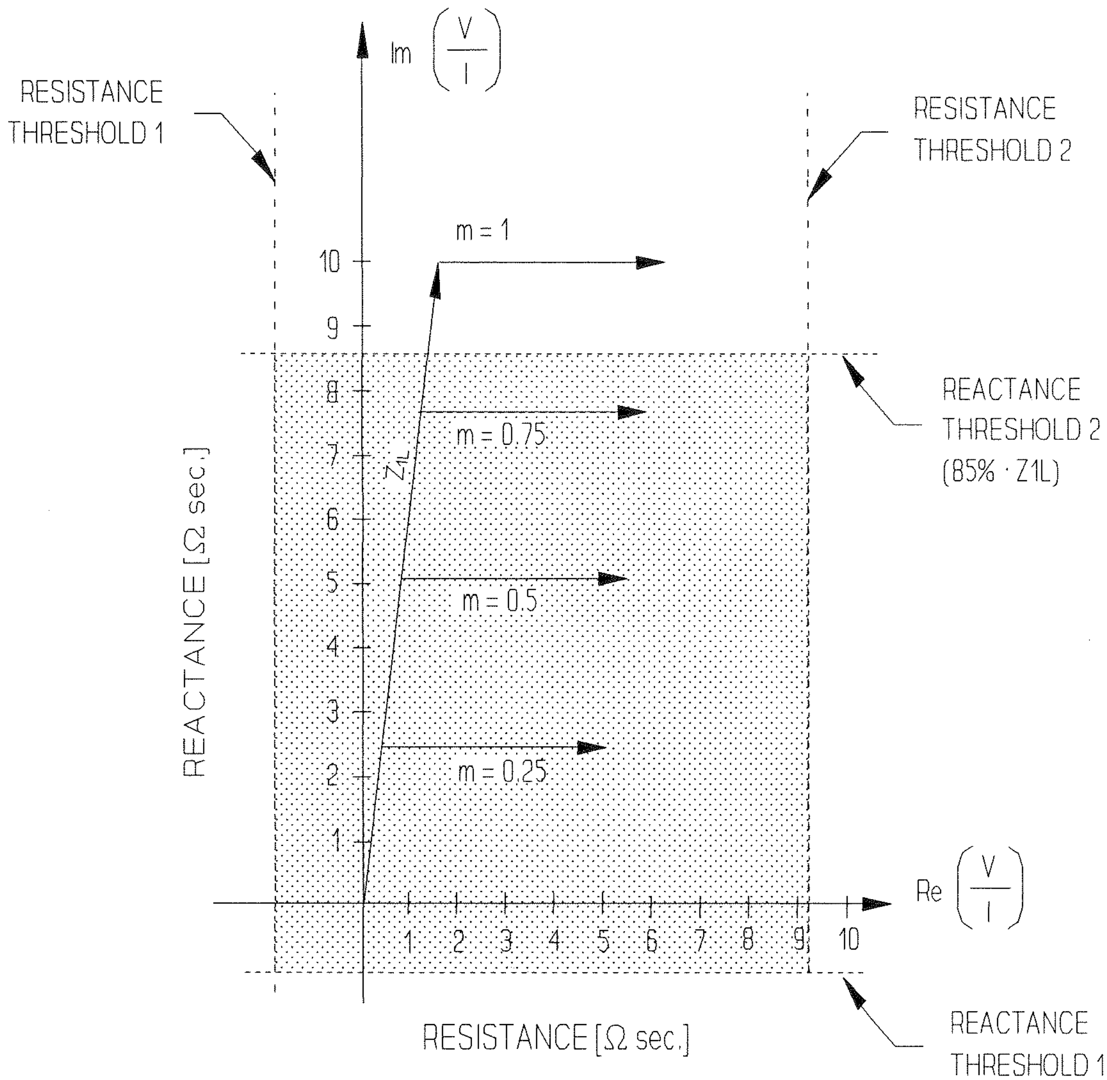


Figure 3: AG Apparent Impedance Compared to Reactance and Resistance Thresholds (Radial Case)

Load Flow and Fault Resistance Effects on Z

What effect does load flow and R_F have on the $Z = V/I$ measurement? Consider the faulted system in Figure 1 again, except now close the switch near Bus R and assume load flows from Bus S to Bus R with $\delta = 30°$.

Figure 4a shows the relay at Bus S overreaches in that Im(V/I) measures a reactance less than the line reactance to the fault. This is because I_F and I are not in phase (Figure 4b) and R_F appears as a complex impedance.

This overreach becomes more pronounced with increasing R_F and δ.

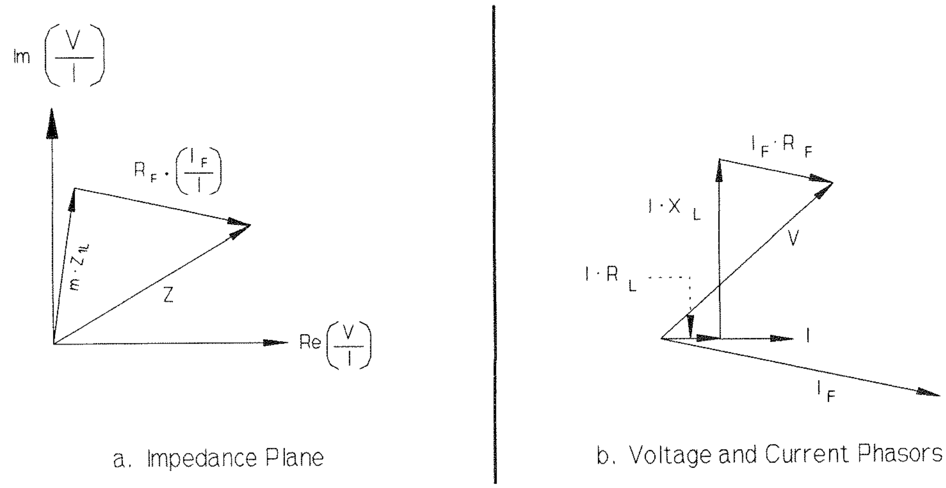


Figure 4: AG Apparent Impedance Overreach with R_F and Load Flow Out (δ = 30°)

Figure 5 illustrates under- and overreaching of the Z = V/I approach for different R_F and load flow conditions. The relay reach is set to $r \cdot Z_{1L}$, where r = 0.85. The Z = V/I approach underreaches for incoming load (I_F leads I), and overreaches for load out (I_F lags I).

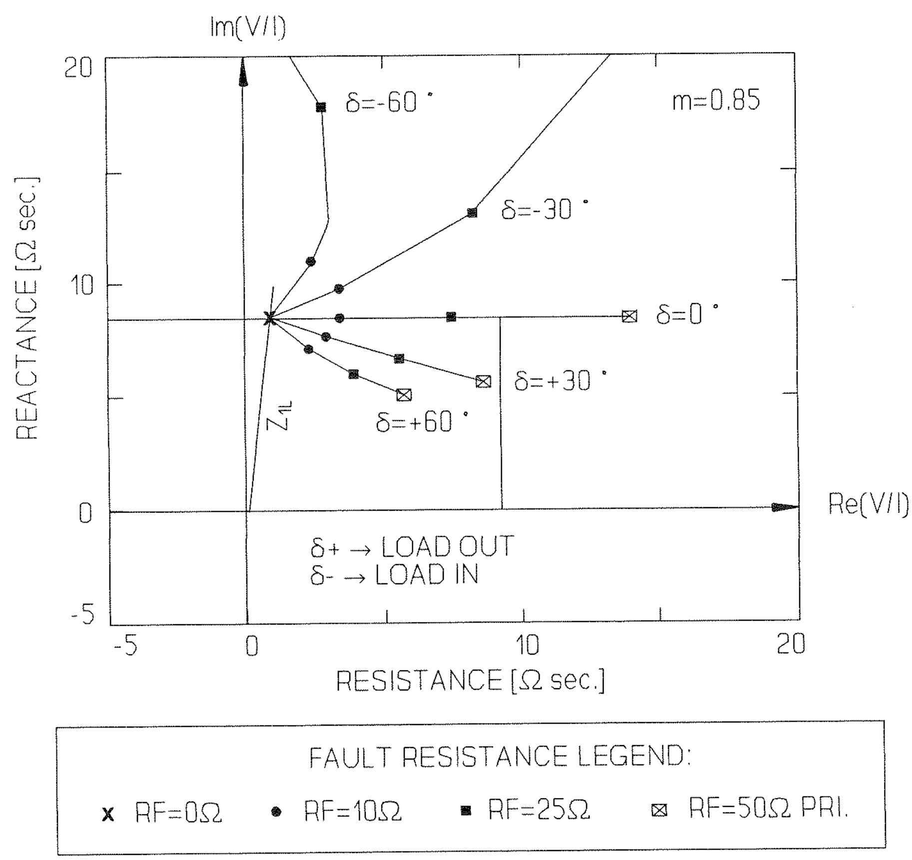


Figure 5: AG Apparent Impedance Method Performance Depends on R_F and δ

To better illustrate the under- and overreaching of this reactance measurement, compare $\underline{x} = \text{Im}(Z)/\text{Im}(r \cdot Z_{1L})$ to unity for the same system conditions shown in Figure 5. The results are shown in Figure 6 where the ideal $\underline{x}$ is unity.

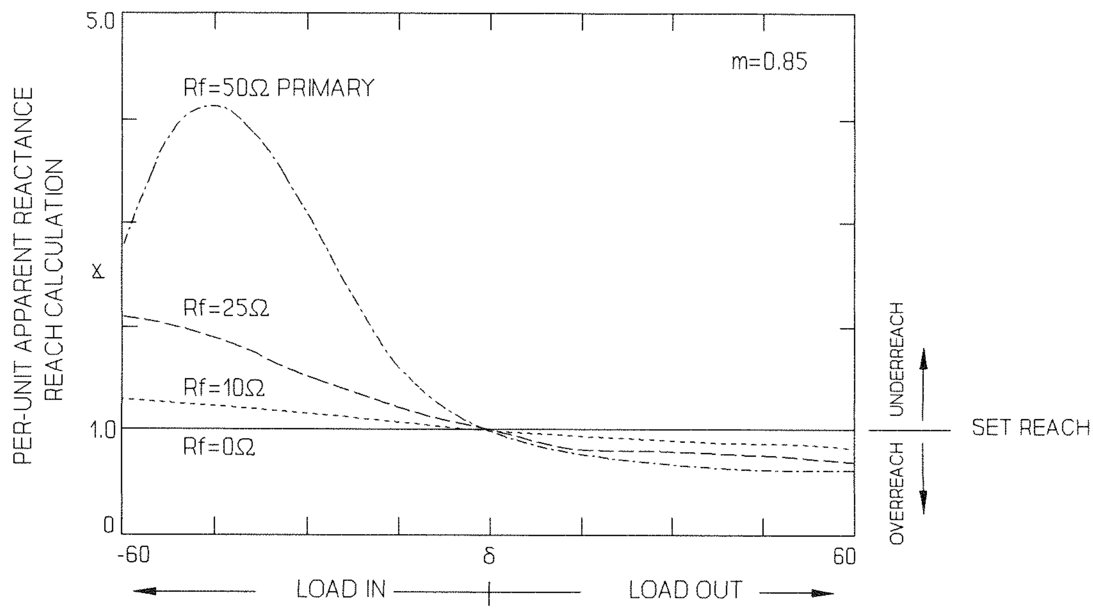


Figure 6: **Per-unit AG Apparent Impedance Calculation ($\underline{x}$) Illustrates the Amount of Under- and Overreach with Fault Resistance and Load Flow**

The overreaching problem can be minimized by restricting the reactive reach. For a zone 1 ground distance element, this means reducing the amount of instantaneous protective coverage. We could also restrict the resistive coverage to avoid the overreach which occurs for line-end faults. However, this penalizes the fault resistance coverage for all fault locations.

IMPROVED QUADRILATERAL CHARACTERISTIC

There are much better ways to estimate the reactance to the fault, and the fault resistance, than to use $R + j \cdot X = V/I$.

These better methods depend on proper selection of polarizing quantitites.

Reactance Element

Reference 1 describes one means of obtaining an improved reactance characteristic using the sine-phase comparator. This comparator measures the angle, Θ, between the operating (S_{OP}) and polarizing (S_{POL}) signals. The torque of this comparator is defined by $\text{Im}(S_{OP} \cdot S_{POL}{}^*)$; where * indicates the complex conjugate. For $0° \leq \Theta \leq 180°$, the characteristic is a straight line, and the torque is positive. The angles $\Theta = 0°$ and $\Theta = 180°$ define the boundary line in the impedance plane.

The reactance element comparator has the following input signals:

$$S_{OP} = \delta V \qquad S_{POL} = I_P$$

where:

$\delta V = (r \cdot Z_{1L} \cdot I - V)$, line-drop compensated voltage
r = per-unit reach
Z_{1L} = positive-sequence line impedance
$I = I_A + k0 \cdot I_R$
$V = A\phi$ measured voltage
I_P = polarizing current

Figure 4 shows that the faulted phase current is not always in-phase with the total fault current, I_F. Thus, phase current makes a poor choice for the polarizing reference signal. Negative-sequence or residual currents are much better choices.

Equation 3 illustrates why I_R is an appropriate polarizing choice. In this equation, the residual current measured at Bus S is expressed in terms of the total fault current, I_F. For systems where the $\angle Z_{0S} = \angle Z_{0L} = \angle Z_{0R}$ (homogeneous systems), the $\angle I_R$ equals the $\angle I_F$ regardless of the load condition or fault resistance magnitude.

$$I_R = \left[\frac{(1-m) \cdot Z_{0L} + Z_{0R}}{Z_{0S} + Z_{0L} + Z_{0R}} \right] \cdot I_F \qquad \text{Equation 3}$$

By selecting $I_P = I_R$, the reactance element measurement is insensitive to load flow conditions. Equation 4 defines the reach of a residual current (I_R) polarized reactance element of reach r for a boundary fault condition.

$$\underline{r} = \frac{\mathrm{Im}(V \cdot I_R^*)}{\mathrm{Im}(I \cdot Z_{1L} \cdot I_R^*)} \qquad \text{Equation 4}$$

In Equation 4, $\underline{r}$ is equal to the reach setting r for a fault on the boundary of a zone. For faults internal to a zone, $\underline{r} < r$.

To show the improved performance of the I_R polarized reactance element for different R_F and load flow conditions, compare the per unit reach result of Equation 4 to unity; where per-unit reach, $r' = \underline{r}/r$. These results are shown in Figure 7.

From the figure, notice this reactance element does not have the under-and overreaching problems we saw earlier with the $Z = V/I$ method.

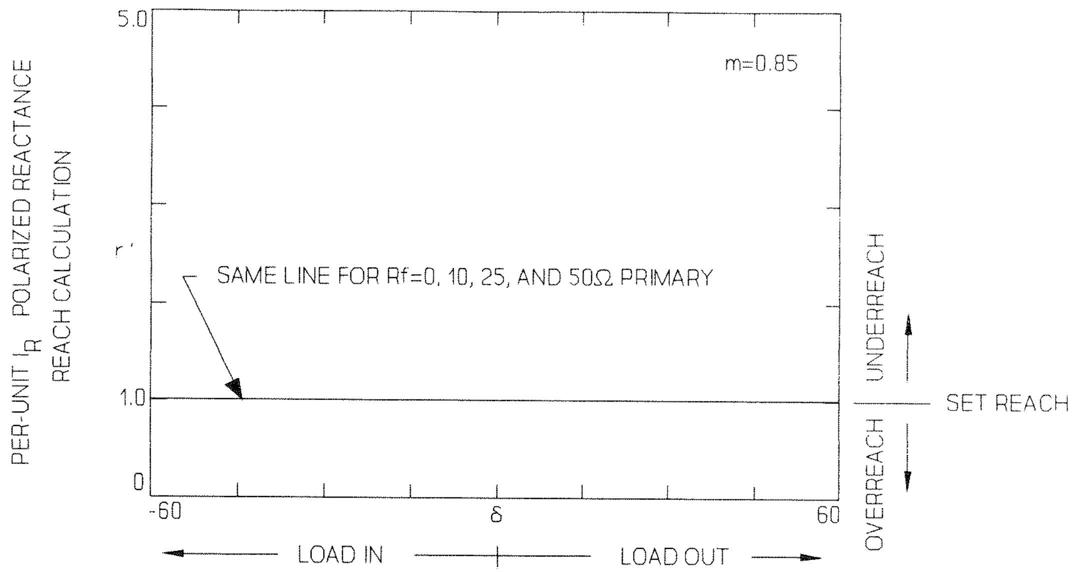


Figure 7: **Per-unit Improved Reactance Reach Calculation (r') Shows I_R Polarization Not Affected by δ and Fault Resistance in Homogeneous Systems**

Resistive Elements

The job of a resistive element is to limit the resistive coverage for a quadrilateral zone of protection. Some resistive elements measure the line resistance plus R_F. Reference 1 describes a resistance element which limits its measurement to R_F alone. Reference 1 also shows the derivation of the following equation for R_F:

$$R_F = \frac{\mathrm{Im}[V\cdot(Z_{1L}\cdot I)^*]}{\mathrm{Im}[(3/2)\cdot(I_2+I_0)\cdot(Z_{1L}\cdot I)^*]}$$

The greatest advantage of this fault resistance element is that its measurement is not appreciably affected by load flow conditions. This allows setting the resistive boundary (or threshold) greater than the minimum load impedance. The resulting quadrilateral characteristic and its response to different load flow and R_F conditions for an AG fault at m = 0.85 is shown in Figure 8.

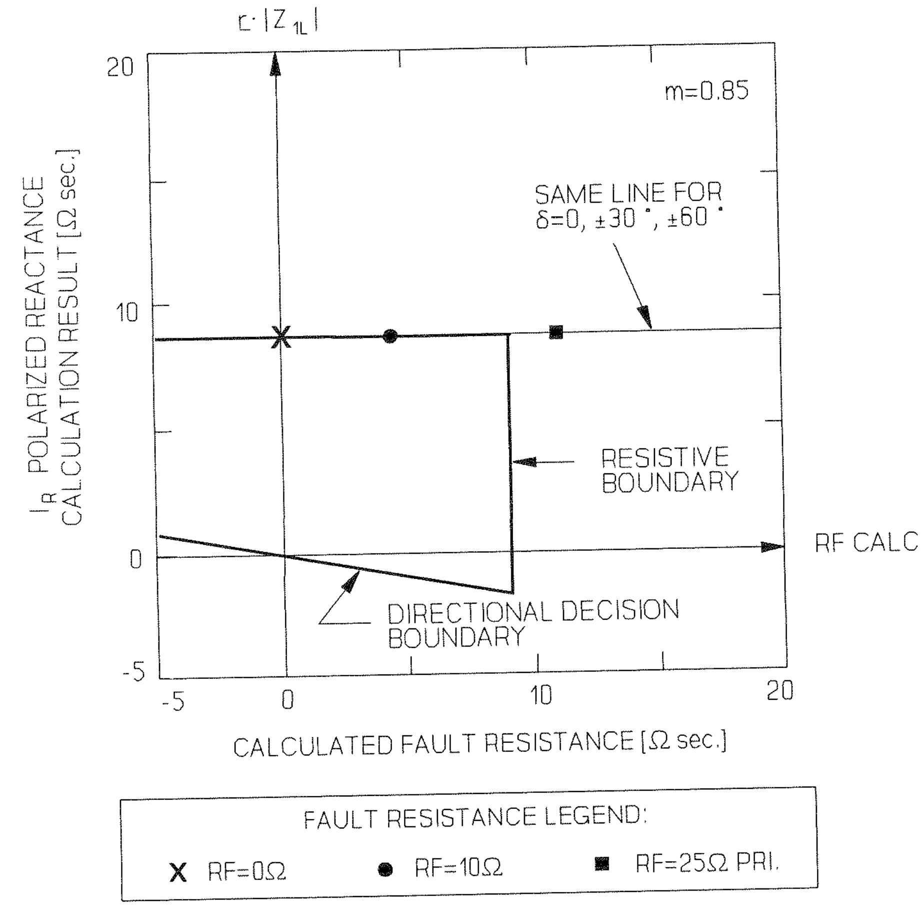


Figure 8: Improved Quadrilateral Characteristic Performance Does Not Depend on δ

MHO CHARACTERISTIC

The typical means of obtaining a mho characteristic is to use the cosine-phase comparator. This comparator measures the phase angle, Θ, between the operating and polarizing signals. For $-90° \leq \Theta \leq 90°$, the characteristic is circular. The angle $\Theta = -90°$ and $\Theta = +90°$ defines the boundary of the characteristic in the impedance plane.

The mho characteristic comparator has two inputs: operating (S_{OP}) and polarizing (S_{POL}):

$$S_{OP} = \delta V \qquad\qquad S_{POL} = V_P$$

where:

$\delta V = (r \cdot Z_{1L} \cdot I - V)$, line-drop compensated voltage
r = per-unit reach
Z_{1L} = positive-sequence line impedance
$I = I_A + k0 \cdot I_R$
V = $A\phi$ measured voltage
V_P = polarizing voltage

Reference 1 describes the torque expression for this cosine-phase comparator, labelled P, as:

$$P = \mathrm{Re}(S_{OP} \cdot S_{POL}^*) = \mathrm{Re}[(r \cdot Z_{1L} \cdot I - V) \cdot V_P^*] \qquad \text{Equation 5}$$

All points where P = 0 define the boundary of a mho characteristic of reach $r \cdot Z_{1L}$.

Self-Polarized Mho Characteristic

Earlier we checked the Z = V/I result against a rectangular characteristic. Let's now test Z against a self-polarized mho characteristic to see how it performs.

For a self-polarized mho characteristic, $V_P = V$. To determine the boundary characteristic of this element, set P = 0, substitute V for V_P in Equation 5, and solve for Z:

$$\mathrm{Re}[(r \cdot Z_{1L} \cdot I - V) \cdot V^*] = 0 \qquad \text{Equation 6}$$

$$\mathrm{Re}(r \cdot Z_{1L} \cdot I \cdot V^*) - |V|^2 = 0$$

Let $r \cdot Z_{1L} = |r \cdot Z_{1L}| \angle \Theta_L$ where Θ_L = positive-sequence line angle
Θ_V = angle of V
Θ_I = angle of I

$$|r \cdot Z_{1L}| = \frac{|V|^2}{\mathrm{Re}\,(I \cdot 1 \angle \Theta_L \cdot V^*)}$$

$$|r \cdot Z_{1L}| = \frac{|V|}{|I| \cdot \cos\,[\Theta_L - (\Theta_V - \Theta_I)]}$$

Next, let $Z = |Z| \angle \phi = V/I$ and $\phi = (\Theta_V - \Theta_I)$

$$|Z| = |r \cdot Z_{1L}| \cdot \cos(\Theta_L - \phi) \qquad \text{Equation 7}$$

When $\phi = \Theta_L$, $|Z| = |r \cdot Z_{1L}|$. Equation 7 describes a mho circle passing through the origin and $r \cdot Z_{1L}$ on the impedance plane (Figure 9). Equation 7 also shows the self-polarized mho distance element is equivalent to testing Z = V/I against a circular characteristic in the impedance plane.

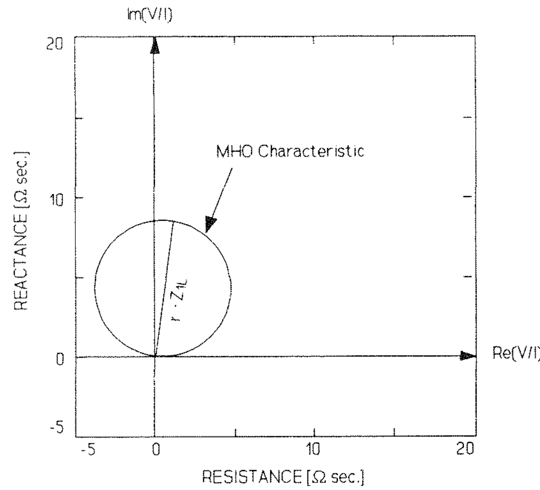


Figure 9: Self-Polarized Mho Characteristic with Reach $r \cdot Z_{1L}$

Figure 10 shows the Z = V/I AG apparent impedance values tested against the self-polarized mho characteristic for different load flow and fault resistance conditions. With load flow and fault resistance, this relay has severe underreaching problems and slight overreaching problems. As compared with the rectangular characteristic, the overreaching problem is reduced because there is less resistive coverage.

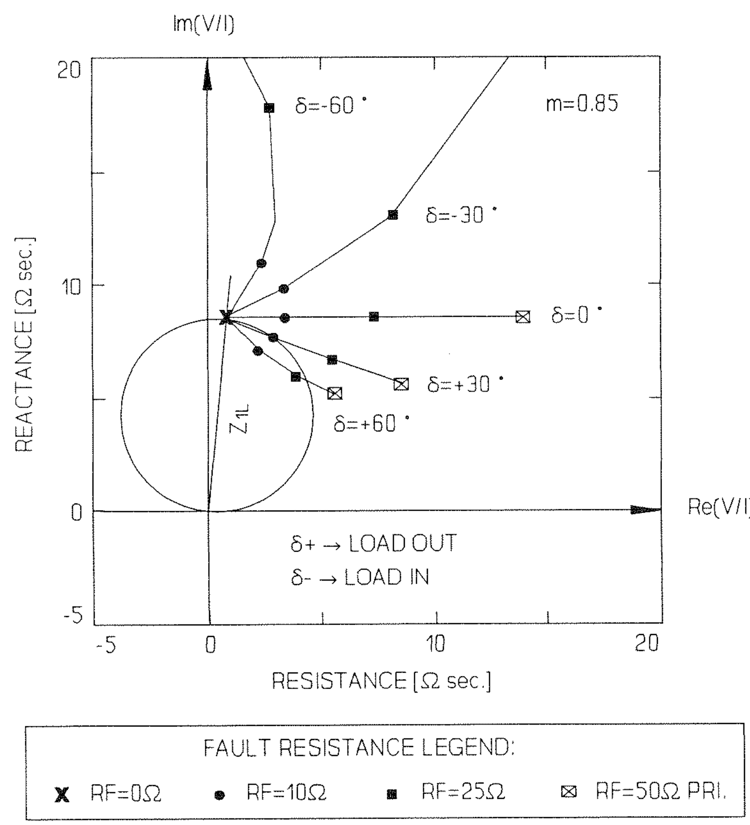


Figure 10: Self-Polarized Mho Performance Depends on R_F and δ

Solving Equation 6 for the minimum reach required to just detect a fault, instead of Z, results in the following expression:

$$\underline{r} = \frac{\mathrm{Re}\ (V \cdot V^*)}{\mathrm{Re}\ (I \cdot Z_{1L} \cdot V^*)}$$

To determine if the fault is inside or outside the characteristic, compare the $\underline{r}$ calculation against r (same process as for the reactance element). The per-unit self-polarized mho element reach result, r', in Figure 11 shows the amount of under- and overreach for various R_F and load flow conditions for the system shown in Figure 1. The source impedance ratio (SIR) for this example equals 0.1 (SIR = $Z_{1S}/(r \cdot Z_{1L})$).

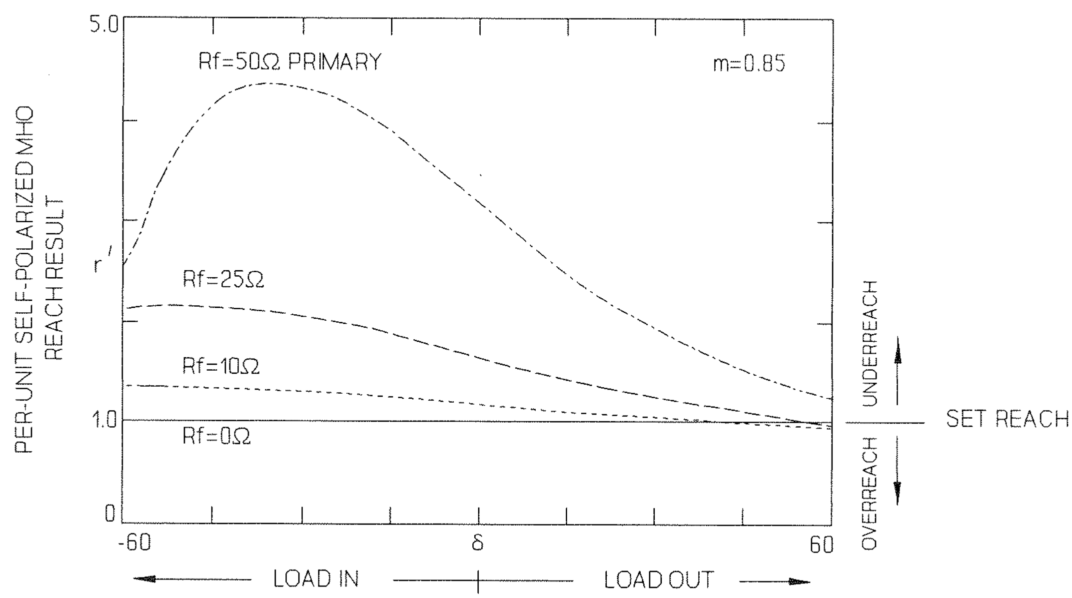


Figure 11: Per-unit Self-polarized Mho Reach Result Illustrates the Amount of Under- and Overreach (for SIR = 0.1)

Positive-sequence Memory Polarized Mho Characteristic

The biggest disadvantage of using V for the polarizing signal is the lack of security for zero voltage faults. For these faults the polarizing signal has no definite phase angle.

The ideal polarizing signal should be available at all times, and not disappear with the fault. One thing we know is that if the system is energized prior to the fault, the prefault voltage is available. We must memorize this prefault voltage and use it for the polarizing signal.

Table 2 compares the self-polarized and phase-voltage memory-polarized cosine-phase-comparator results for an AG fault at m = 0. Notice the $\Theta_{\delta V} - \Theta_{VPRE}$ is available (not indeterminate) and less than 90° for all cases. This shows memorized prefault voltage is excellent polarization.

Table 2: Memory Polarization is Better than Self-Polarization for Close-in Faults

δ	R_F [Ω pri]	Self-Polarized $\Theta_{\delta V} - \Theta_V$	Memory-Polarized $\Theta_{\delta V} - \Theta_{VPRE}$
0°	0	Indeterminate	0.00°
0°	1	85.20°	3.80°
0°	5	86.80°	18.00°
0°	10	88.90°	33.30°
0°	15	90.80°	45.30°
30°	0	Indeterminate	2.40°
30°	1	87.60°	6.10°
30°	5	89.10°	20.30°
30°	10	90.90°	35.30°
30°	15	92.60°	47.10°

If we use memorized positive-sequence voltage ($V1_{mem}$) for V_P, the polarizing signal is present even if the memory has expired for all fault types except three-phase faults. Another benefit of $V_P = V1_{mem}$ is the improved security during open-pole conditions in single-pole tripping schemes.

To obtain the boundary equation for the positive-sequence memory-polarized mho characteristic, set $V_P = V1_{mem}$ in Equation 5, and solve for the boundary condition. This yields:

$$\underline{r} = \frac{\text{Re}\,(V \cdot V1_{mem}^{\ *})}{\text{Re}\,(I \cdot Z_{1L} \cdot V1_{mem}^{\ *})} \qquad \text{Equation 8}$$

Figure 12 shows the per-unit reach calculation (r') for different R_F and load flow conditions for a SIR = 0.1. Notice there is not much difference between the performance of the $V1_{mem}$ and V polarized mho elements for AG faults at m = 0.85. This is due to the relatively strong source behind the relay. What happens if we increase the SIR?

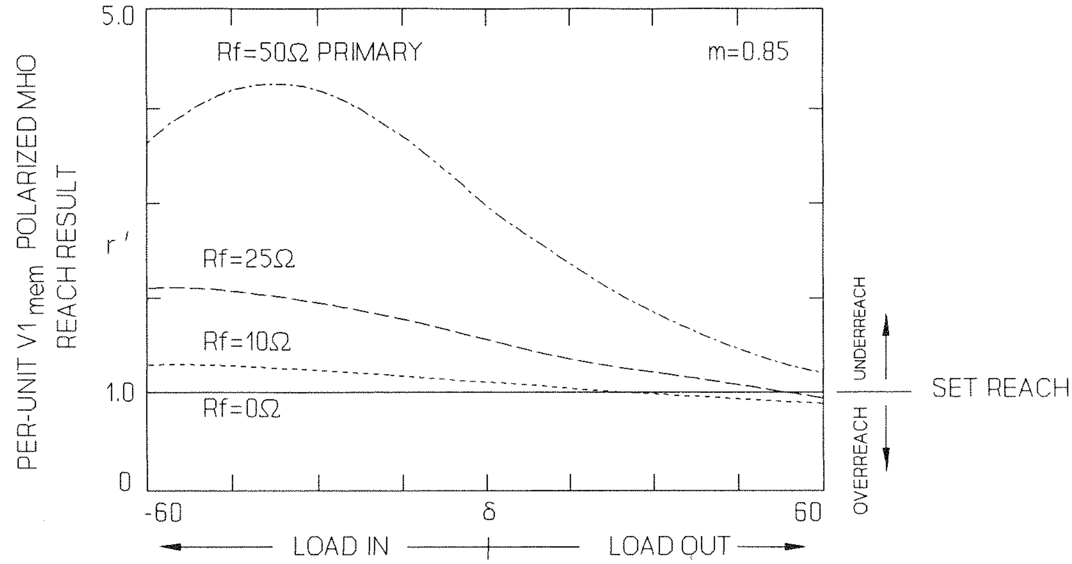


Figure 12: Per-unit $V1_{mem}$ Polarized Mho Reach Result Illustrates the Same Amount of Under- and Overreach as the Self-Polarized Mho (for SIR = 0.1)

Figures 14 and 15 show the amount of underreach for the self- and positive-sequence memory polarized relays for a AG fault at m = 0.85 on the system in Figure 13 (SIR = 1.2). From these figures, you can see the $V1_{mem}$ relay performance is better than the self-polarized relay because its underreach is much less. This improvement increases with increasing SIR.

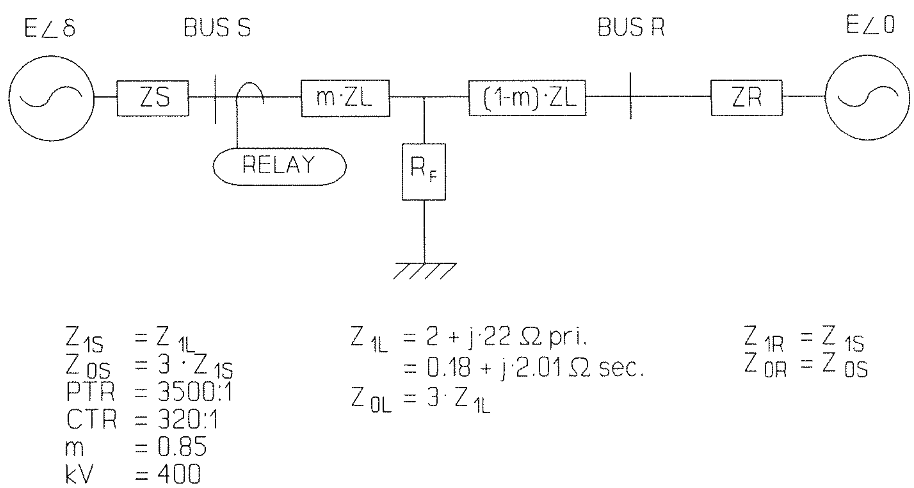


Figure 13: System Single Line Diagram (SIR = 1.2)

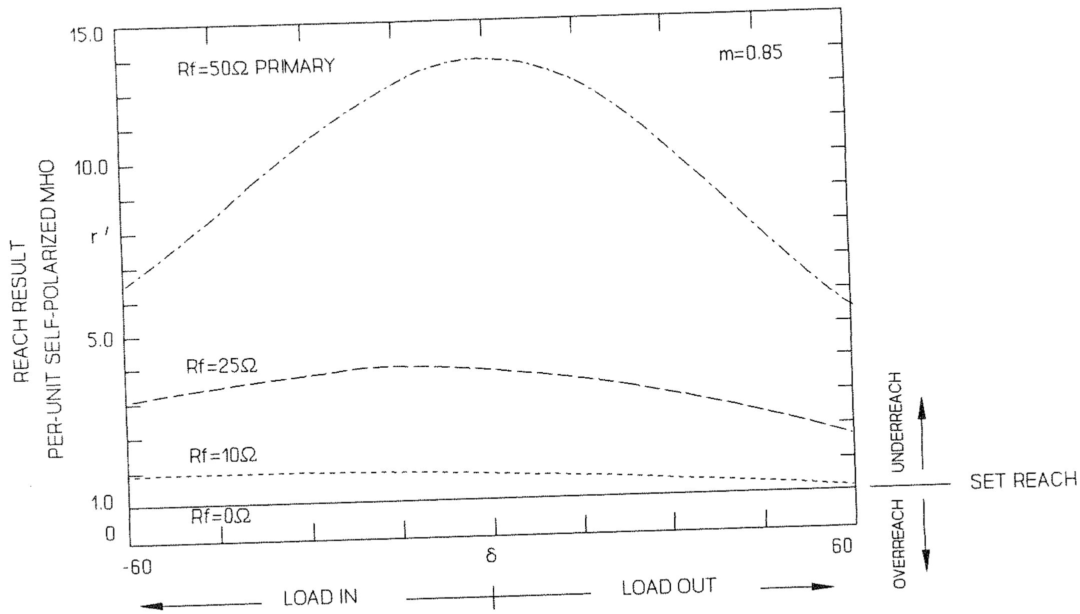


Figure 14: **Self-Polarized Mho Underreach Increases Dramatically with Increasing SIR (SIR shown = 1.2)**

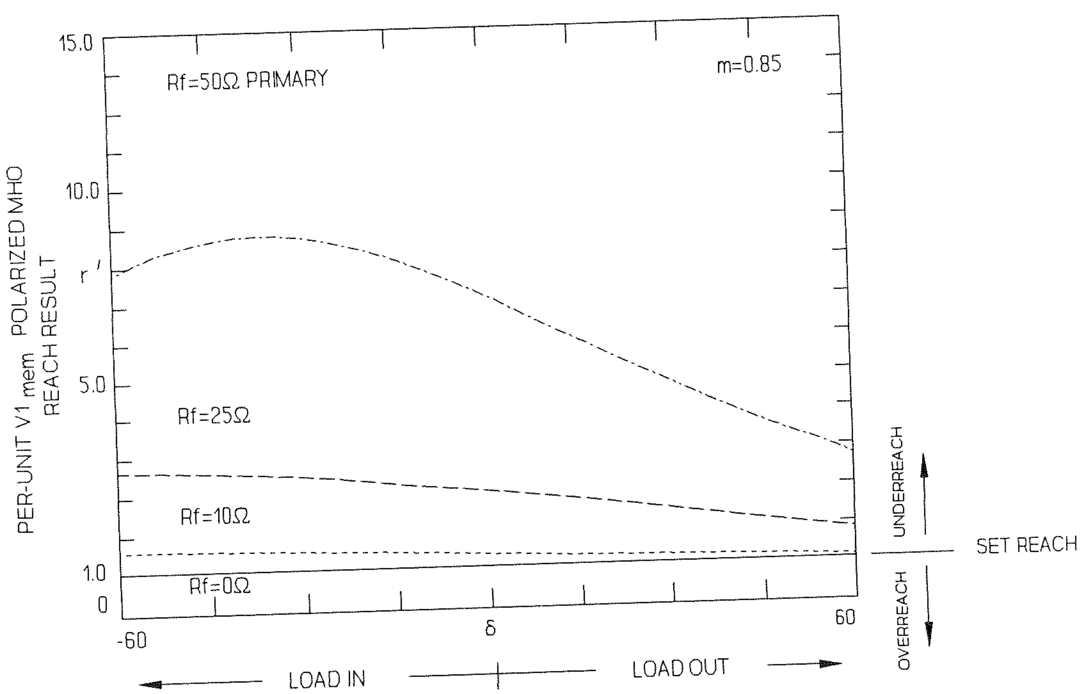


Figure 15: **$V1_{mem}$ Polarized Mho Underreach is Less than Self-Polarized Mho with Increasing SIR (SIR shown = 1.2)**

THE $V1_{mem}$ POLARIZED MHO DENOMINATOR TERM IS A DIRECTIONAL ELEMENT

In Equation 8, V in the numerator could be zero for close-in faults. Therefore, we cannot rely on the sign of r to dependably indicate direction. Fortunately, the denominator of Equation 8 defines a directional element. This directional element measures the angle Θ between the operate signal ($I \cdot Z_{1L}$) and the polarizing signal ($V1_{mem}$). The angle $\Theta = -90°$ and $\Theta = 90°$ defines the zero torque threshold.

Label the denominator term for the AG mho distance element as MAGD. From Equation 8, MAGD is defined as:

$$MAGD = Re[I \cdot Z_{1L} \cdot (V1_{mem})^*] \qquad \text{Equation 9}$$

Figure 16 shows the inputs to this denominator for forward (16a) and reverse (16b) AG faults. For simplicity of illustration, this example assumes $\delta = 0°$, $R_F = 0$, and a 90° system angle.

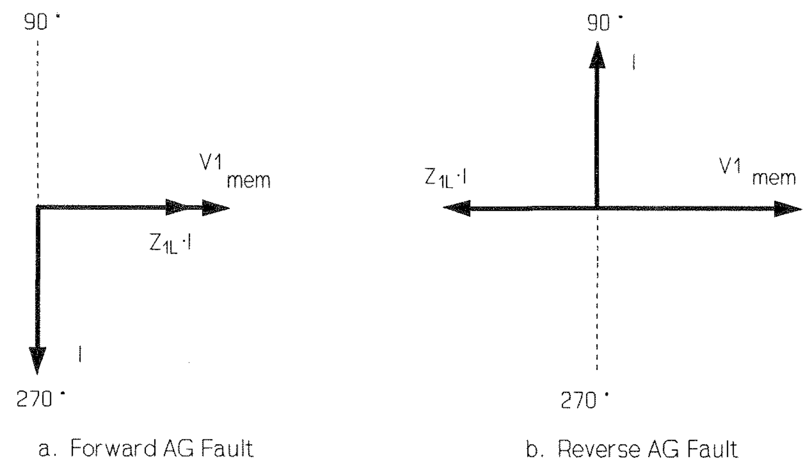


Figure 16: AG Mho Denominator Term Inputs for Forward and Reverse AG Faults

The sign of MAGD is positive for forward faults and negative for reverse faults. Because the sign changes with fault direction, we can use the denominator as a directional element.

CAN SUPERPOSITION IMPROVE THE Z = V/I PERFORMANCE?

The Z = V/I method under- and overreaches for resistive ground faults with load because I and I_F are not in phase. Subtracting balanced prefault current (I_{APRE}) from the faulted phase current (I_A) results in a current (I') which is in phase with the total fault current, I_F, assuming balanced prefault load. This I' current is referred to as the superposition current. Let's analyze the approach of substituting I' for the faulted phase current in the AG equation shown in Table 1 and check its performance.

Figure 17 shows the superposition current apparent impedance calculation (Z') results for AG faults at m = 0.85 and various R_F and load flow conditions. This modification of the apparent impedance relay has severe under- and overreaching tendencies. One difference between the distance measurement results shown in Figure 17 and those shown in Figure 10, is the load flow conditions which cause under- and overreach are exactly opposite. The superposition apparent impedance relay underreaches for load out, and overreaches for incoming load flow, where the simple apparent impedance relay underreached for incoming load and overreached for load out.

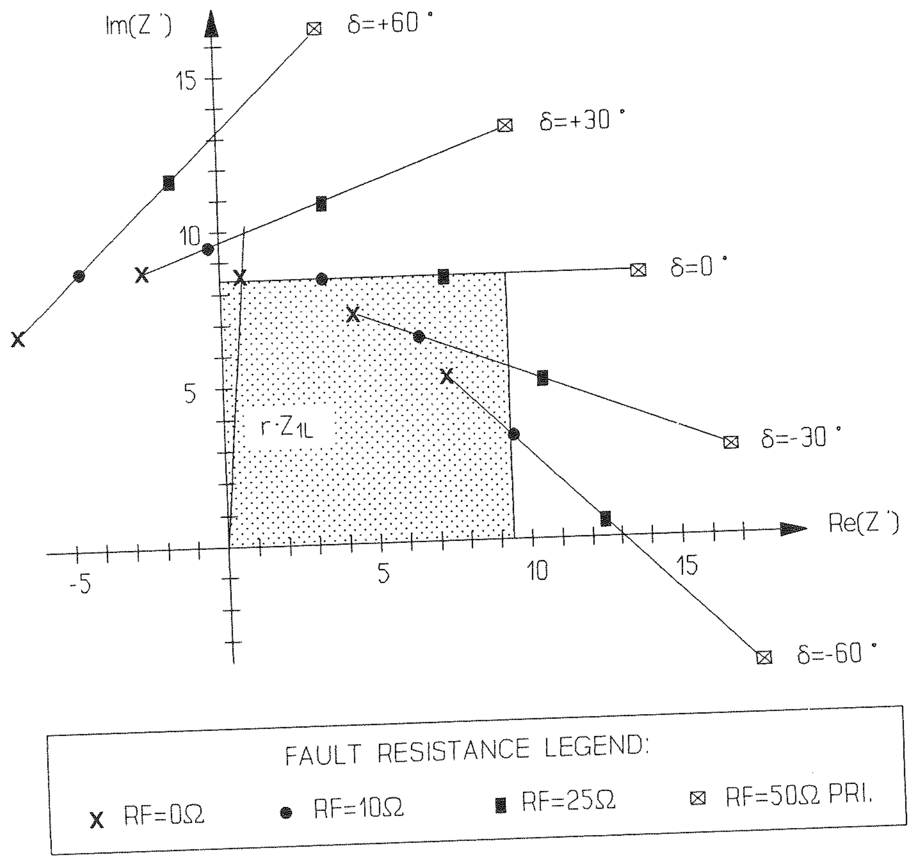


Figure 17: Superposition Apparent Impedance Performance Depends on R_F and δ

Also notice the right-hand resistance threshold must be set almost ten times greater than the line resistance value to just detect an AG fault with zero fault resistance during heavy incoming load flow conditions (δ = -60°). While it is desirable to detect these faults, this large resistance reach makes the relay susceptible to overreach for lower incoming load conditions (eg. δ = 30°) with fault resistance.

These last points, coupled with those noted earlier for the Z = V/I calculation results, emphasizes that an apparent impedance relay is not an acceptable alternative to properly polarized distance elements.

WHEN CAN WE USE V/I CHARACTERISTICS?

One example where Z = V/I can be useful is the detection of balanced conditions, such as load. For example, we can define incoming ($ZLOAD_{IN}$) and outgoing ($ZLOAD_{OUT}$) load regions with thresholds and check the V/I calculation result against these thresholds. These regions define the Load-Encroachment Characteristics. If the calculated V/I is inside either of these regions, the load-encroachment logic blocks the phase distance elements from tripping. A load condition, a three-phase mho (3P21) and relay load-encroachment characteristics are shown in Figure 18. For this heavy load-out condition, the 3P21 element is blocked from operating because load also resides inside the $Load_{OUT}$ load-encroachment characteristic.

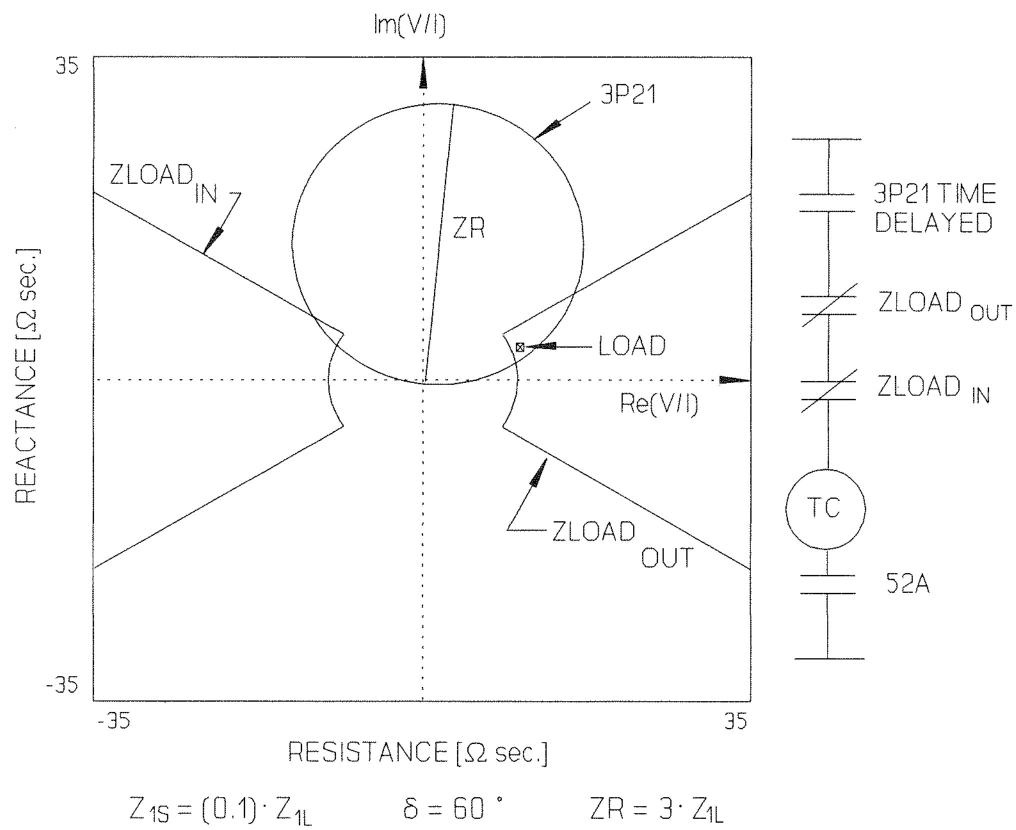


Figure 18: Z = V/I Load-Encroachment Characteristic Prevents Phase Distance Element from Operating Under Heavy Load Conditions

SUMMARY

1. The apparent impedance calculation result (Z = V/I) compared with geometric characteristics (box or circle) presents serious under- and overreaching problems when load flow and fault resistance are combined.

2. The Z = V/I method has the same performance deficiencies as self-polarized relays.

3. The Z = V/I approach is not improved by using superposition current.

4. Writing the torque equation for a properly polarized relay characteristic, setting it equal to zero, and solving for the minimum reach required to just detect the fault yields a result which we can test against scalar thresholds (Reference 1). This method achieves the very desirable result of offering one calculation (per fault loop) for all zones with the performance of properly polarized relay elements.

5. We present ground-fault-resistance elements which estimate the fault resistance. This estimate rejects the load-influenced positive-sequence current, can be used for multiple zones, and can be set greater than the minimum load impedance.

6. The Z = V/I calculation can be used to detect load conditions, as in the new load-encroachment characteristic (Reference 1).

In conclusion, the Z = V/I calculations have some use in elements intended to measure load, but perform poorly compared to properly polarized relay element calculations, as represented by the equations given for $\underline{r}$ and R_F.

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