Zero Sequence Compensation (k_0) Factors for Ground Distance Relaying

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Abstract

Ground distance relaying relies on the computation of a positive sequence equivalent representing the ground fault impedance. The positive sequence equivalent is computed at the relay location by the equation: $Z_1 = V_A/(I_A + 3I_0k_0)$ for a ground fault on phase A. The constant, k_0, is known as the zero sequence compensation factor. The zero sequence compensation factor is often defined as $k_0 = (Z_0 - Z_1)/3Z_1$ for the homogeneous line, where Z_0 and Z_1 are the zero and positive sequence line impedances. However, zero sequence effects can cause the equivalent Z_1 to be different from that computed for phase faults requiring an adjustment to k_0 in order to obtain the desired reach. Zero sequence effects can be defined, in general, as differences in the zero sequence distribution factors from the fault location to relay location as compared to the positive sequence distribution factors. These effects include the presence of auto-transformers, mutual coupling, current flows beyond the remote bus and shunt devices.

The zero sequence compensation factor can be computed for a specific fault location by the equation: $k_0 = (V_A/Z_1 - I_A)/3I_0$ for a ground fault on phase A. The value of Z_1 corresponds to the positive sequence reach entered in the relay for the zone element being studied. Zero sequence effects typically cause an increase in the equivalent positive sequence impedance requiring an increase in k_0 in order to prevent under reaching. Many relays offer a single k_0 setting and combined phase/ground zone reaches. It is up the relay engineer to find the best compromises considering all system conditions and the effect of k_0 on fault location. Relays with multiple k_0 settings and separate phase and ground distance reaches offer more choices. This paper also provides an example of k_0 calculations with and without zero sequence effects.

Basic Distance Element Operating Equations

Modern micro-processor based relays sample analog voltage and current waveforms which are digitized and fed through comparators in order to determine whether operation is required. Operating and polarizing methods vary but most devices rely on zone distance reaches based on positive sequence equivalents. Figure 1 shows a typical impedance network.

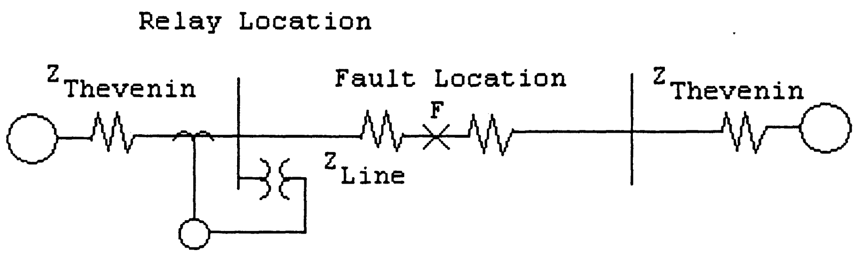

Figure 1

The basic method for phase distance measurements is to use delta voltages and currents. That is, $\mathbf{(V_A\text{-}V_B)/(I_A\text{-}I_B)}$, $\mathbf{(V_B\text{-}V_C)/(I_B\text{-}I_C)}$ and $\mathbf{(V_C\text{-}V_A)/(I_C\text{-}I_A)}$.

Consider the following faults:
(Note: Z_L is taken to be the impedance of the line between the relay location and the fault and could be expressed as mZ_L, where m equals the fraction of the total line distance between the relay location and the fault and Z_L equals the total line impedance. All quantities are solved for at the relay location.)

<u>Three-Phase Fault</u>

$V_A = V_B = V_C = 0$ at the fault for a bolted fault
$I_{A0} = I_{A2} = 0$

So, $V_{A1} = 0 + I_{A1}Z_{L1}$ $\quad I_A = I_{A1}$
$V_{B1} = a^2I_{A1}Z_{L1}$ $\quad I_B = a^2I_{A1}$
$V_{C1} = aI_{A1}Z_{L1}$ $\quad I_C = aI_{A1}$

then $(V_A\text{-}V_B)/(I_A\text{-}I_B) = (1\text{-}a^2)I_{A1}Z_{L1}/(1\text{-}a^2)I_{A1} = Z_{L1}$

The result is the same for all phase pairs.

Phase-to-Phase Fault

Consider a Phase B-C bolted phase-to-phase fault:

$I_{A0} = 0$
$V_{A1F} = V_{A2F}$
$I_{A2} = -I_{A1}$

Let $Z_{L1} = Z_{L2}$

$V_{A1} = V_{A1F} + I_{A1}Z_{L1}$ where V_{A1F} equals the positive sequence voltage at the fault
$V_{A2} = V_{A2F} + I_{A2}Z_{L2}$ where V_{A2F} equals the negative sequence voltage at the fault
$= V_{A1F} - I_{A1}Z_{L1}$

$V_A = 2V_{A1F}$
$V_B = a^2V_{A1} + aV_{A2} = a^2(V_{A1F} + I_{A1}Z_{L1}) + a(V_{A1F} - I_{A1}Z_{L1})$
$= (a^2 + a)V_{A1F} + (a^2 - a)I_{A1}Z_{L1}$
$V_C = aV_{A1} + a^2V_{A2} = (a^2 + a)V_{A1F} + (a - a^2)I_{A1}Z_{L1}$
$V_B - V_C = 2(a^2 - a)I_{A1}Z_{L1}$

$I_B = a^2I_{A1} + aI_{A2} = a^2I_{A1} - aI_{A1} = (a^2 - a)I_{A1}$
$I_C = aI_{A1} + a^2I_{A2} = (a - a^2)I_{A1}$
$I_B - I_C = 2(a^2 - a)I_{A1}$

Then, $(V_B - V_C)/(I_B - I_C) = 2(a^2 - a)I_{A1}Z_{L1}/2(a^2 - a)I_{A1} = Z_{L1}$

The problem faced was to develop an equation that allowed for distance measurements of ground faults. Since V_0 is the greatest at the fault, a calculation of V_0/I_0 at the relay location would not yield the zero sequence line impedance to the fault [1]. The equation used to calculate a positive sequence equivalent for ground faults can be derived by considering a bolted single-line-to-ground fault applied to Phase A in figure 1:

Single-Line-to-Ground Fault

$V_{A0F} = V_{A0} - I_{A0}Z_{L0}$
$V_{A1F} = V_{A1} - I_{A1}Z_{L1}$
$V_{A2F} = V_{A2} - I_{A2}Z_{L2}$

$V_{A0F} + V_{A1F} + V_{A2F} = 0$
Let $Z_{L2} = Z_{L1}$

Then $V_{A0} + V_{A1} + V_{A2} - (I_{A0}Z_{L0} + I_{A1}Z_{L1} + I_{A2}Z_{L2}) = 0$
and $V_{A0} + V_{A1} + V_{A2} = V_A = (I_{A0}Z_{L0} + I_{A1}Z_{L1} + I_{A2}Z_{L1}) = I_{A0}Z_{L1} + I_{A1}Z_{L1} + I_{A2}Z_{L1} - I_{A0}Z_{L1} + I_{A0}Z_{L0}$

Since $I_{A0} + I_{A1} + I_{A2} = I_A$ then,
$V_A = I_A Z_{L1} + I_{A0}(Z_{L0} - Z_{L1})) = Z_{L1}(I_A + I_{A0}(Z_{L0}/Z_{L1} - 1))$

So, $Z_{L1} = V_A/(I_A + I_{A0}(Z_{L0}/Z_{Li} - 1)) = V_A/(I_A + I_{A0}(Z_{L0} - Z_{L1})/Z_{L1}) = V_A/(I_A + 3I_{A0}(Z_{L0} - Z_{L1})/3Z_{L1})$

This defines equation (1) for the positive sequence equivalent for a ground fault.

(1) $\mathbf{Z_1 = V_A/(I_A + 3I_0k_0)}$ for Phase A where k_0 = zero sequence compensation factor

The zero sequence compensation factor is defined by:

(2) $\mathbf{k_0 = (Z_0 - Z_1)/3Z_1}$

Equation (1) is often used by ground mho type distance elements, in an operating form and in conjunction with polarization, to determine whether operation is required for ground fault conditions even though k_0 often reflects only the sequence self impedances of the line. Equation (2) would be accurate for all circumstances, if Z_0 represented an effective zero sequence impedance analogous to the affect of infeed on positive sequence impedances [2]. Various forms of equation (2) have been used by different manufacturers with the above being fairly common.

The phase distance equations (delta voltages and currents) are applicable for three-phase, phase-phase and phase-ground faults.

The ground distance equations (for each phase) are applicable for phase-ground and phase-phase-ground faults.

There are many excellent texts and papers which detail specific measurement and digital techniques including various operating and polarizing quantities and the functions of comparators and dynamic response [3], [4], and [5].

Equations Solved for k_0

Exact Equation for k_0

When equation (1) is solved for k_0

(3) $\mathbf{k_0 = (V_A/Z_1 - I_A)/3I_0}$

This is the exact form for k_0 for any particular fault location for which the positive sequence impedance is known. The positive sequence impedance corresponds to the relay setting for the zone distance element being studied, or the positive sequence impedance to some compromise location such as the remote bus, and can already include the effects of positive sequence infeed.

Homogeneous Line Approximation

A homogeneous line is considered to be a line with consistent tower type and conductor type. In fact, in the case of ground distance relaying it is more correct to say, that as far as k_0 values are concerned, a homogeneous line is one where the zero sequence current distribution factors remain unchanged from the fault back to the relay location.

Equation (2) defines k_0 for the homogeneous line condition:

$\mathbf{k_0 = (Z_0 - Z_1)/3Z_1}$

A typical value of k_0 based on equation (2) for a 115-kV transmission line with overhead ground wires is:

$k_0 = 0.766$ for $Z_0 = 3.3Z_1$

Zero Sequence Effects

Equation (2) is most often used or recommended for calculating k_0. Unfortunately, when used with just the line parameters, it ignores any zero sequence effects which alter the zero sequence currents seen at the relay location. Zero sequence effects include zero sequence sources such as auto-transformers, mutual coupling, and zero sequence current distribution beyond the remote bus.

These effects alter the zero sequence current seen at the relay location and the value of k_0 required to reach to the fault location is different than that implied by equation (2) which only considers sequence self impedances.

Consider the presence of an auto-transformer on the line:

Then for a single-line-to-ground fault,

- V_A at the relay location is increased due to more current to the fault

- I_0 at the relay location is decreased due to zero sequence masking

- I_A at the relay location is then also reduced

and therefore k_0 must be increased in order to keep the positive sequence equivalent correct. The effective Z_0 is therefore increased.

With too small a k_0 value, the relay will under reach the desired zone reach assuming that the ground distance positive sequence settings equal the phase distance positive sequence settings.

Classes of Relays

Relays which utilize equation (1) can be organized into different classes for the purpose of considering settings options:

- Relays which have one k_0 setting and single positive sequence zone settings for each combined phase/ground distance zone element.

- Relays which have no k_0 setting, with k_0 calculated internally by equation (2) from entered line parameters, and single positive sequence zone settings for each combined phase/ground distance zone element.

- Relays which have multiple k_0 settings and separate positive sequence zone settings for each individual phase and ground distance zone element.

Examples of k_0 Calculations

Homogeneous Line with no Zero Sequence Effects

Consider the system in Figure 2.

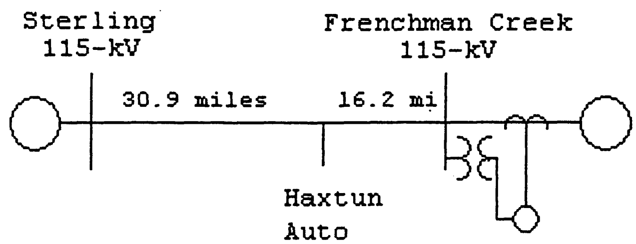

Figure 2

The line impedances are as follows:

$Z_{L1} = 0.0774 + j0.2708 = 0.2816$ @ 74 deg pu on 100 MVA base
$Z_{L0} = 0.2784 + j0.8862 = 0.9289$ @ 72.6 deg pu on 100 MVA base

Per equation (2) k_0 is calculated as:

$k_0 = (Z_0 - Z_1)/3Z_1 = 0.766$ @ -2.1 degrees

This value, using equation (2), applies all along the line.

A fault study was performed and k_0 calculated using equation (3) for a zone 1 fault at 90% to Sterling with the relay location considered at Frenchman Creek. The auto-transformer was off-line for comparison. The results are described below in figure 3.

The k_0 values using equations (2) and (3) agree for this case since there were no zero sequence effects.

Calculation of Zero Sequence Compensation Factor

Enter values from fault study as magnitude in per unit and angle in degrees

Input Data

	Magnitude (pu)	Angle (degrees)
V_A	0.576	-2.1
Z_1	0.2534	74
I_A	1.235	-75
$3I_0$	1.356	-75.3

Enter Case description here. Frenchman Creek - Sterling 115-kV
Calculated for SLG 90% to Sterling
No auto at Haxtun

Output Data

	Magnitude (pu)	Angle (degrees)
V_A/Z_1	2.273	-76.1
$V_A/Z_1 - I_A$	1.039	-77.4
k_0	0.766	-2.1

This spreadsheet uses the following equation for k_0:

$k_0 = (V_A/Z_1 - I_A)/3I_0$

Figure 3

Line with Auto-transformer at Tap Location

The situation is the same as above but with an auto-transformer at the tap location. There is no positive sequence source at the tap. Figure 4 lists the results.

Calculation of Zero Sequence Compensation Factor

Enter values from fault study as magnitude in per unit and angle in degrees

Input Data

	Magnitude (pu)	Angle (degrees)
V_A	0.592	-2.2
Z_1	0.2534	74
I_A	1.182	-74.4
$3I_0$	1.143	-73

Enter Case description here.
Frenchman Creek - Sterling 115-kV
Calculated for SLG 90% to Sterling
With auto at Haxtun

Output Data

	Magnitude (pu)	Angle (degrees)
V_A/Z_1	2.336	-76.2
$V_A/Z_1 - I_A$	1.155	-78.0
k_0	1.011	-5.0

This spreadsheet uses the following equation for k_0:

$k_0 = (V_A/Z_1 - I_A)/3I_0$

Figure 4

The proper value for k_0 has risen to 1.011. If a ground distance relay at Frenchman Creek were to use the value of 0.766, the relay would require a positive sequence equivalent impedance of 0.288 per unit instead of 0.2534 per unit in order to operate. The error is approximately 13% and the relay would fail to see the fault in zone 1 if the ground distance zone 1 reach was the same as or matched the phase distance zone 1 reach.

If the relay being utilized has multiple k_0 values and separate positive sequence zone settings for each individual phase and ground distance zone element, then k_0 for zone 1 could be set to 0.766 per line parameters and the ground distance zone 1 setting adjusted for the proper reach. Figure 5 below show the results derived from a fault study for the adjusted zone 1 reach. These settings would allow accurate fault determination up to the location of the auto-transformer.

Calculation of Equivalent Positive Sequence Reach

Enter values from fault study as magnitude in per unit and angle in degrees

Input Data

	Magnitude (pu)	Angle (degrees)
V_A	0.592	-2.2
I_A	1.182	-74.4
k_0	0.766	-2.1
$3I_0$	1.143	-73

Enter Case description here
Frenchman Creek - Sterling 115-kV
Calculated for SLG 90% to Sterling
With auto at Haxtun
k_0 set per line parameters

Output Data

	Magnitude	Angle
	(pu)	(degrees)
Z_1	0.288	72.5

This spreadsheet uses the following equation for Z_1:

$$Z_1 = V_A/(I_A + 3I_0k_0)$$

Figure 5

Effect of k0 on Fault Location

Single end fault location methods are effected by the value of k_0 used. Double end methods can be done in any sequence. In the example with the auto-transformer above, the k_0 value calculated from the line parameters would yield accurate fault locations up to the tap. The value of k_0 calculated at the zone 1 reach would yield accurate fault location in the vicinity of the zone 1 reach.

For relays which have separate positive sequence reaches for the phase and ground distance zone elements, the reach of the ground distance elements can be adjusted to account for zero sequence effects instead of adjusting k_0.

Relay Settings: Options and Suggestions

The options available for adjusting ground distance elements will depend on the class of the relay involved.

For a relay with one k_0 setting and single positive sequence zone settings for the combined phase/ground distance zone elements, the relay could be set as follows:

- Calculate k_0 for a remote bus or zone 1 fault and set zones as usual (e.g. zone 1 = 90%, etc.)

or

- Set k_0 per line geometry and accept compromises in ground distance reaches

For a relay with k_0 internally computed from the line parameters and single positive sequence zone settings for the combined phase/ground distance zone elements, the relay could be set as follows:

- Calculate k_0 for a remote bus or zone 1 fault and adjust the zero sequence line impedance by using $Z_0 = Z_1(3k_0 + 1)$ and set zones as usual

or

- Use normal line impedances and accept compromises in ground distance reaches

For relays with multiple k_0 settings and separate positive sequence zone settings for the individual phase and ground distance zone elements, the relay could be set as follows:

- Use k_0 computed from line geometry for zone 1 and adjust zone 1 ground distance setting as necessary to obtain desired reach. The reach can be calculated by the use of a fault study and equation (1): $Z_1 = V_A/(I_A + 3I_0k_0)$. This allows accurate single end fault location up to the point of the first zero sequence effect.

For zones 2 and 3 it may be advantageous to use k_0 calculated for a remote bus or zone 2 fault and adjust the zone element settings as necessary to obtain the desired reaches.

For faults beyond the remote bus, the zero sequence currents often distribute somewhat differently than the positive sequence currents. This tends to create the necessity for increasing k_0 values as the fault location becomes more remote from the relay location. In general, any value of computed k_0 is accurate only for the specific system conditions or fault location for which it was calculated.

When computing k_0, the value of Z_1 used in equation (3) should be the positive sequence reach entered in the relay for the zone element in question or the positive sequence impedance to some compromise location such as the remote bus.

Conclusions and Recommendations

Zero sequence effects can cause ground distance elements to over or under reach the desired location unless care is taken in the choice of k_0 and the positive sequence reaches. The equation for k_0 given by equation (3) yields accurate values with minimal effort utilizing a fault study.

Zero sequence compensation factor values tend to increase as the fault location becomes more remote from the relay location. Too low a k_0 value causes the ground distance relay reach to be short. It is up to the protection engineer to study various fault locations and system conditions and a choose k_0 value(s) that represents the best overall choice(s).

References

[1] Blackburn, J. L., *Protective Relaying - Principles and Applications*, Marcel Dekker, Inc., New York and Basel, 1987.

[2] ABB, *Protective Relaying Theory and Applications*, Marcel Dekker, Inc., New York, Basel, Hong Kong, 1994.

[3] GEC, *Protective Relays - Application Guide*, GEC Measurements, Stafford, England, 1987.

[4] Roberts, Jeff and Schweitzer, E.O., *Distance Relay Element Design*, Proceedings of the 46th Annual Conference for Protective Relay Engineers, Texas A&M University, College Station, Texas, April 12-14, 1993

[5] Guzman, A., Roberts, J. and Schweitzer, E.O., *Z = V/I Does Not Make A Distance Relay*, Proceedings of the 20th Annual Western Protective Relay Conference, Spokane, Washington, October 19-21, 1993.