

Computer Simulation Of Current Transformers And Relays  
For Performance Analysis

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Presented Before The  
14th Annual Western Relay Conference  
Spokane, Wa.  
October 20-23, 1987

# Computer Simulation Of Current Transformers And Relays For Performance Analysis

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## Introduction

Present switchgear standards [1] allow the use of low ratio, low accuracy class current transformers in switchgear. In these applications the available primary fault currents can exceed more than 200 times the ct rating producing grossly distorted secondary currents due to the saturation of the cts magnetic core. Figure 1 shows an example of the distorted output waveform produced by a 150/5 ct subjected to a 40 ka offset primary current. The secondary current exhibits a high magnitude current pulse for less than 4 ms.in each half cycle. With this current waveform the operation of conventional overcurrent relays is not specified and is affected by armature saturation and eddy currents due to the fast current rise.



Figure 1. Waveforms of 150/5 ct subjected to 40 KA  
Primary current with offset

With the wide spread use of these cts in switchgear, it is not unexpected that anomalous behavior of overcurrent relays has been experienced. Unfortunately, the standards do not address relay performance. Although electromechanical overcurrent relays are widely used, methods are not generally available to predict their performance under such severe conditions. Consequently, without full scale testing the manufacturer cannot guarantee operation.

Digital filtering, the process of combining current samples from an A/D converter during a fixed period, is used in several microprocessor based relays to extract digital quantities representing phasor components. High frequency harmonics are eliminated from the measurement by an anti-aliasing filter preceding the converter. In the development of a new overcurrent relay, it was conjectured that the anti-aliasing filter alleviates the effects of ct saturation.

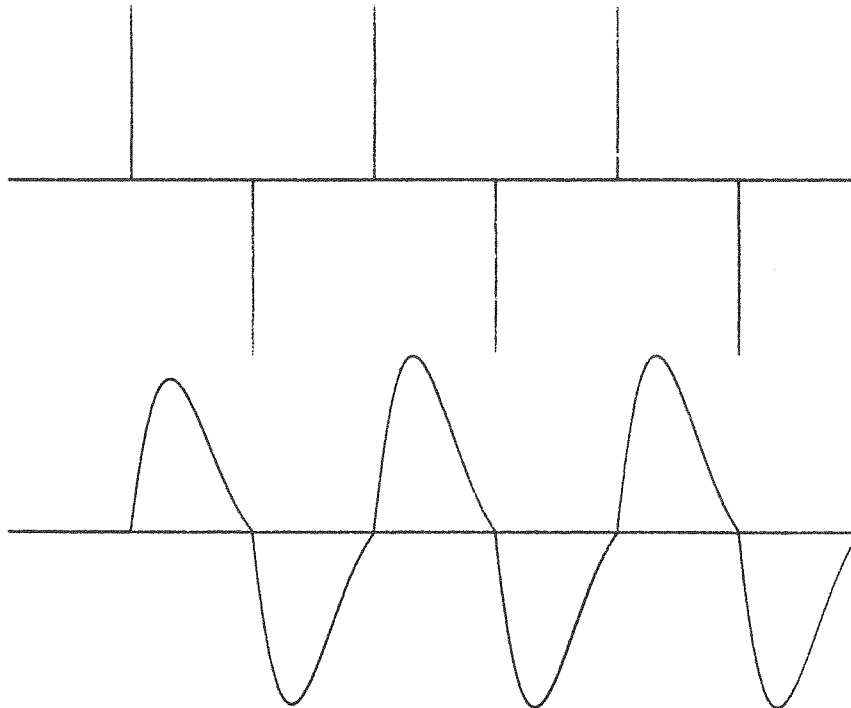


Figure 2 Two pole filter response to a series of impulses occurring each half cycle

The filter is usually a two pole design with a cutoff frequency near the fundamental frequency. Consequently, its response to the high current pulse output of a saturated ct approaches its impulse response which is the sine-like waveform shown in Figure 2. During extreme saturation digital sampling of this waveform provides the input quantities for executing the relay algorithm.

Testing the relay and cts at extreme current levels was deemed too costly and time consuming for evaluating the response of various filter designs. Fortunately, computer implementation of modern analytical techniques provide an affordable alternative. This paper describes how computer simulation was used to "test" the performance of the relay and how the computed performance was subsequently verified by full scale tests in the high current laboratory.

#### Description of Relay Under Test

The subject relay is basically a general purpose three phase and ground overcurrent relay for switchgear application. However, being a fully microprocessor based design, it incorporates a wide range of tap settings, with five standard inverse and definite time characteristics. Since the unit also incorporates metering, communications port, a detailed event record, and self diagnostics it has been given the acronym 'IMPRS' for Integrated Microprocessor Protective Relay System.

In the IMPRS each phase and ground current is sampled four times per cycle and converted to a digital quantity representing the magnitude of the current. The algorithm used has proved ideal for protective relay applications since it has little response to dc offset and rejects even harmonics. However, its performance together with the anti-aliasing input filter remained to be demonstrated with severely distorted current of saturated cts.

Figure 3 is a schematic diagram of its major elements and their correspondence with the computer simulation. The diagram shows a primary ct connection to the circuit of the relay. In the relay an input transformer feeds secondary current to a burden resistor. The voltage signal developed across the burden feeds a low-pass anti-aliasing filter (LPF). The filter output is then routed in sequence to a programmable gain amplifier (PGA) along with the other phase and ground current signals (not shown) by means of the multiplexer (MUX). The PGA provides software control of the relay tap setting. The A/D converter then converts the PGA analog output to the digital quantity used by the processor.

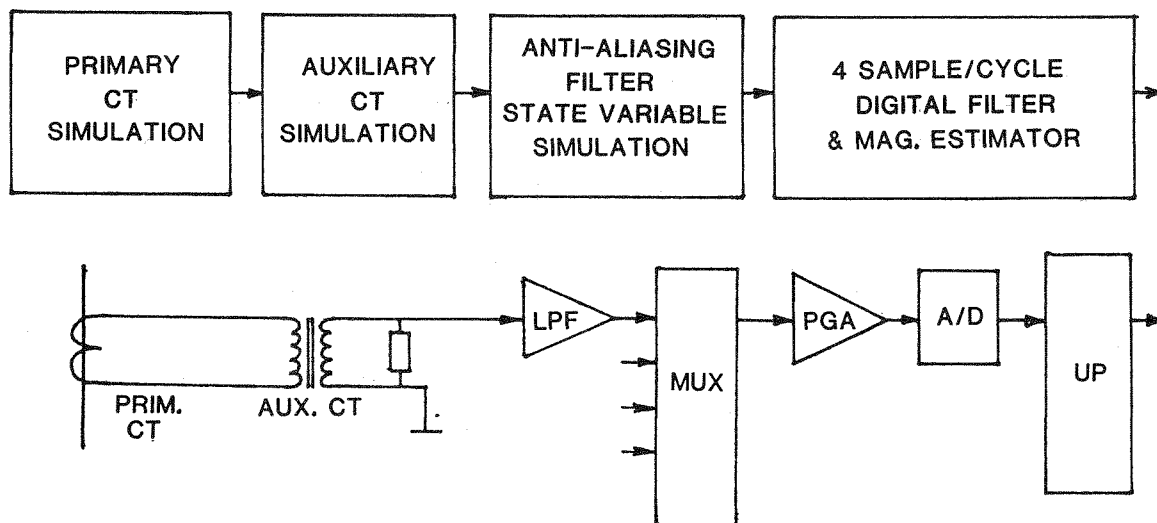


Figure 3. IMPRS schematic and block diagram of simulation

### Simulation

Verifying performance with low ratio cts and extremely high fault currents presents a problem because the resulting waveforms can only be obtained using a full scale short circuit generator. While full scale tests are required for qualification, they are too costly and time consuming for the investigation of a wide range of ct ratings, fault currents, and circuit conditions. At the same time, the functions of the microprocessor based relay are easily formulated. Consequently, the computer simulation of the relay system described below was used to verify performance.

In the simulation, the ct and functional components of the relay system were modeled by separate computer routines executed at discrete time intervals, each model providing the input data for the next model in sequence as shown by the block diagram in Figure 3. The computer models provided quantitative evaluation for:

- (a) The non-linear behavior at high transient currents in the primary and auxiliary current transformers.
- (b) The response of the anti-aliasing filter to the distorted current signal.
- (c) The transient performance of the 4-sample amplitude estimator and the time overcurrent relay algorithm.

## The CT Model

The simple equivalent circuit shown in Figure 4 is the basis of the ct model. In this circuit the non-linear inductance,  $L_m$ , represents the varying magnetizing inductance of the ct during large current excursions.  $L_m$  is calculated from the relation:

$$L_m = \mu_0 \mu_r N_2^2 A_c / l_c \quad (1)$$

where:

- $\mu_r$  = relative incremental permeability of the ct core material.
- $\mu_0$  = permeability of free space.
- $N_2$  = number of secondary turns.
- $A_c$  = cross sectional area of the ct core.
- $l_c$  = mean length of the flux path in the ct core.

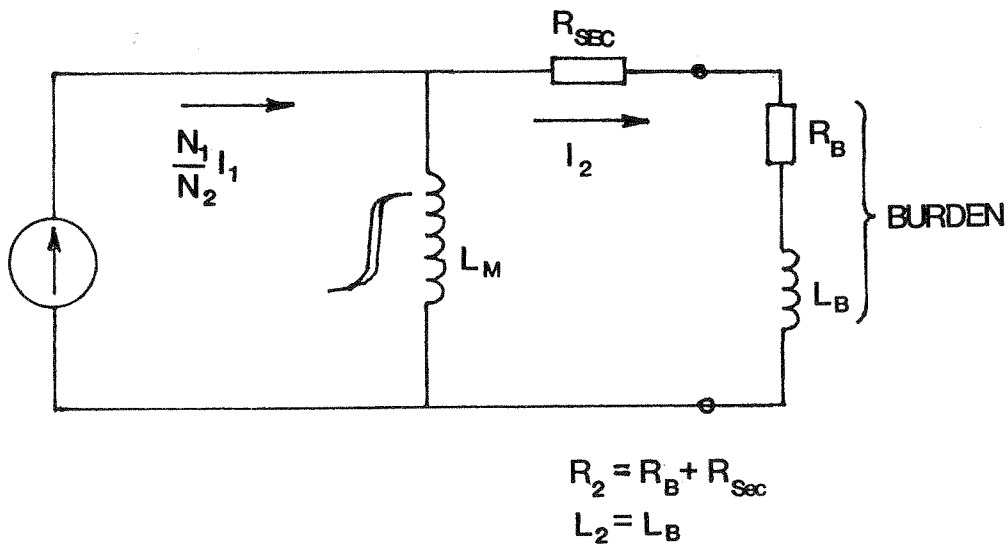


Figure 4. Equivalent circuit of current transformer

The variable core permeability is the slope of the B/H curve of the core material at any specific point:

$$\mu_o \mu_r = dB/dH \quad (2)$$

The circuit model needs a defined relationship between B and H so the permeability can be calculated. Touhy and Panek [3] have used one form of the Frolich equation [2] to estimate transient overvoltages produced in power transformers by current chopping. The same form is used in simulating the circuit's B/H curve:

$$B = H/(c+b|H|) \quad (3)$$

where b and c are constants determined by the core material. The core permeability, being the slope of the B/H curve is expressed:

$$\mu_o \mu_r = dB/dH = (1-b|B|)^2/c \quad (4)$$

When the core flux density reaches the saturation flux density, B(sat), the relative permeability of the core approaches unity. This provides one condition defining how the constants b and c are related.

The maximum slope of the B/H curve occurs at B=0. This is the maximum permeability of the core material,  $\mu_i$ , and is a known constant. Therefore set B=0 in equation (4) and solve for c:

$$c = 1/\mu_o \mu_i \quad (5)$$

Then using this value for c and B = B(sat) solve for b:

$$b = (1-1/\sqrt{\mu_i})/B(\text{sat}) \quad (6)$$

Using these constants the Frolich equation now defines the B/H curve between B=0 and B=B(sat). The model disregards the equation for flux densities greater than B(sat) and fixes  $\mu_o$  as the permeability. Figure 5 shows the resulting B/H curve which is a smooth single-valued curve emulating the B/H relation over a wide range of values. Although the effects of hysteresis and eddy currents are not modelled by the Frolich equation, the single valued curve model proved entirely adequate for the simulation.

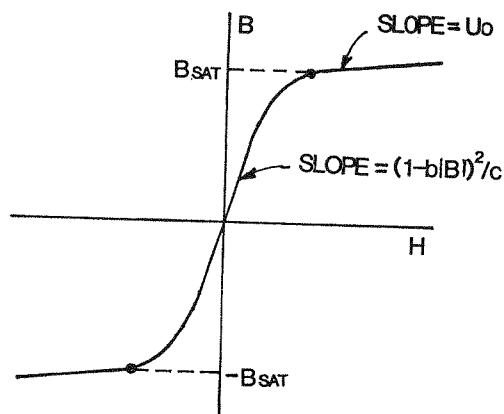


Figure 5. B/H curve emulated using the Frolich Equation

The circuit equations describing the relation of the ct secondary current,  $I_2$ , and the core flux density,  $B$ , to the primary current  $I_1$  is:

$$N_2 A_C dB/dt = L_m [(N_1/N_2) dI_1/dt - dI_2/dt] \quad (7)$$

$$dI_2/dt = (N_2 A_C / L_2) dB/dt - (R_2 / L_2) I_2 \quad (8)$$

Ordinary numerical techniques can be used to solve for the secondary current taking into account the dependence of the core permeability on flux density. However, the second order PS method [4] was found to be very stable over the wide range of coefficients encountered allowing the use of larger time increments to reduce computation time.

#### The Anti-aliasing Filter Model

The response of the anti-aliasing filter to the discrete secondary current values provided by the ct model was determined using the state variable method similar to that described by Gill [5]. The method defines each derivative of the system as an output state. The highest derivative is assigned the value  $X_0$  which is then intergrated to obtain the next state  $X_1$  and in general any state is found by integrating the next lowest state.



This is done by the program using the trapezoidal rule:

$$X(I)_n = X(I)_{n-1} + 0.5[X(I-1)_n + X(I-1)_{n-1}]TD$$

where I indicates the state and n the period of integration.

The Transform of the filter is :

$$E0/E1 = K/(S^2+aS+b) \quad (9)$$

where S is the Laplace operator and K,a,and b are the coefficients which determine the filter response characteristics. The state equation is derived from the transform by cross multiplying and solving for the highest order of S:

$$S^2E0 + aSE0 + bE0 = KE1$$

$$S^2E0 = KE1-(aSE0 + bE0) \quad (10)$$

Since the operator S indicates the process of differentiation:

$$X0 = KE1-(aX1-bX2) \quad (11)$$

where:  $X0 = dx1/dt = d^2E0/dt^2$   
 $X1 = dx2/dt = dE0/dt$   
 $X2 = E0$

The state diagram of equation (10) shown in Figure 6 gives an over view of the process.

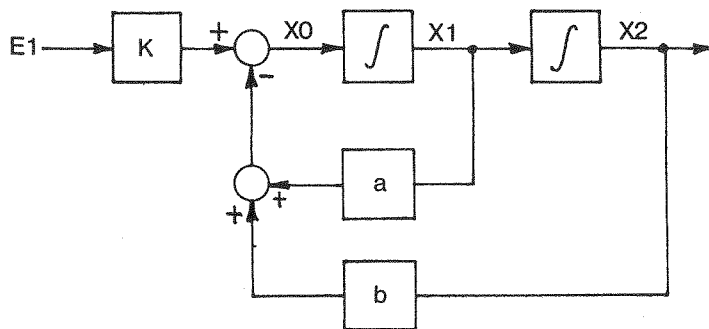


Figure 6. State diagram of two-pole filter

## Digital Filter and Magnitude Estimator

The computer uses the direct implementation of the relay algorithms to process the filter model output. Figure 7 shows a sine wave output sampled at a 240 hertz rate of four samples per cycle. Sample S1 is taken at an arbitrary phase angle and sample S2 90 degrees later in time. By definition S1 and S2 are sine and cosine of an arbitrary phase angle and therefore fully define the phase and magnitude of the sampled sine wave.

In a digital filter more than just the two basic samples per cycle are combined to obtain a more desirable frequency response. The digital filter used combines four samples per cycle as follows in order to reject dc offset and even harmonics:

$$S1 - S2 - S3 + S4 = 2\sqrt{2} \sin(\phi + 45^\circ) \quad (12)$$

$$S2 - S3 - S4 + S5 = 2\sqrt{2} \cos(\phi + 45^\circ) \quad (13)$$

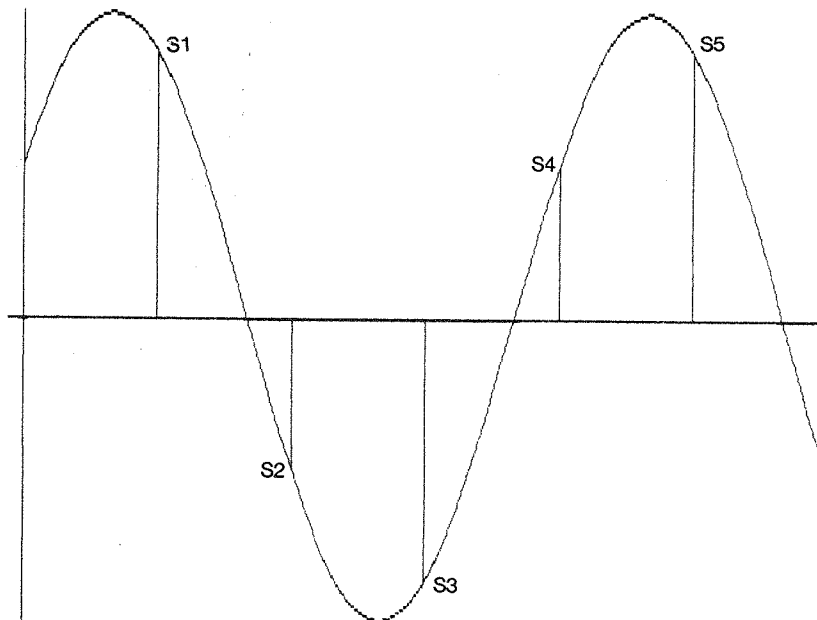


Figure 7. Sampling the current signal

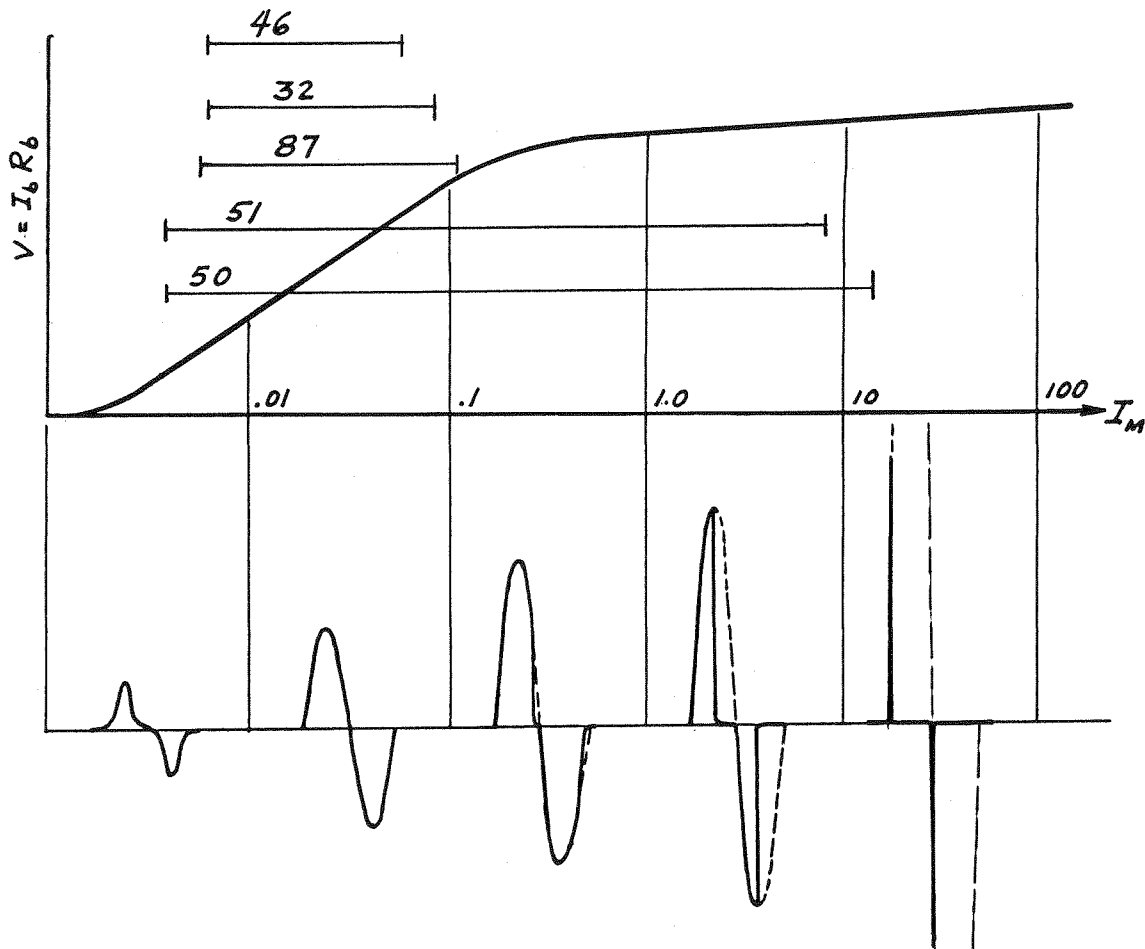


Figure 8. CT magnetizing curve showing range of operation and related waveforms.

### Regions of Operation

Figure 8 shows the range of secondary current waveforms related to the magnetization curve of a ct. The figure also shows that most relays require sine wave input with application confined to the linear region of the saturation curve. However, since the standards allow low ratio, low accuracy cts, overcurrent relays are inadvertently expected to operate over the entire range of waveforms shown in Figure 8.

In the IMPRS unit each phase and ground current input is sampled and converted to a digital number representing the current magnitude. The maximum bit count of the A/D converter is assigned to a perunit current equal to the range of the time-current characteristics. The linear range of the input transformer and filter is this range multiplied by the range of tap settings including that of the instantaneous element. Consequently, in the linear region, the A/D converter produces a digital number proportional to the input current. Currents beyond this range produce the maximum digital output.

Considering the range of application shown in Figure 8, it is important that all currents beyond the linear range produce the maximum digital output, especially the short duration, high magnitude pulses produced by severely saturated cts. This was predicted by the computer simulation and verified by high current tests.

#### Test Program and Results

The unit was tested with a range of cts from 50/5 to 2000/5 with primary currents from 200 to 40,000 amps. In all cases the unit under test produced nominal trip times for currents within the linear range of the A/D. All currents beyond the linear range produced the minimum trip time predicted by the corresponding simulation case. Table 1 gives instantaneous trip test results.

Table 1. IMPRS High Current Test Results

CT	Amps.	Fault	Offset/Sym	M	Trip Time(Ms.)
50/5	210	P-P	0	4.20x	40(Pickup)
	1,086	P-P	0	21.7x	21
	4,670	P-P	0	93.4x	21
	10,000	P-P	0	200x	30
150/5	760	P-P	0	5.06x	23
	9,920	P-P	0	66.1x	23
	40,400	P-P	0	269x	24
	46,200	P-P	0	308x	29
400/5	2,000	P-P	S	5x	30
1200/5	10,300	3-P	N/A	8.58x	29
2000/5	25,600	3-P	N/A	12.5x	26
	39,500	P-P	0	19.8x	26

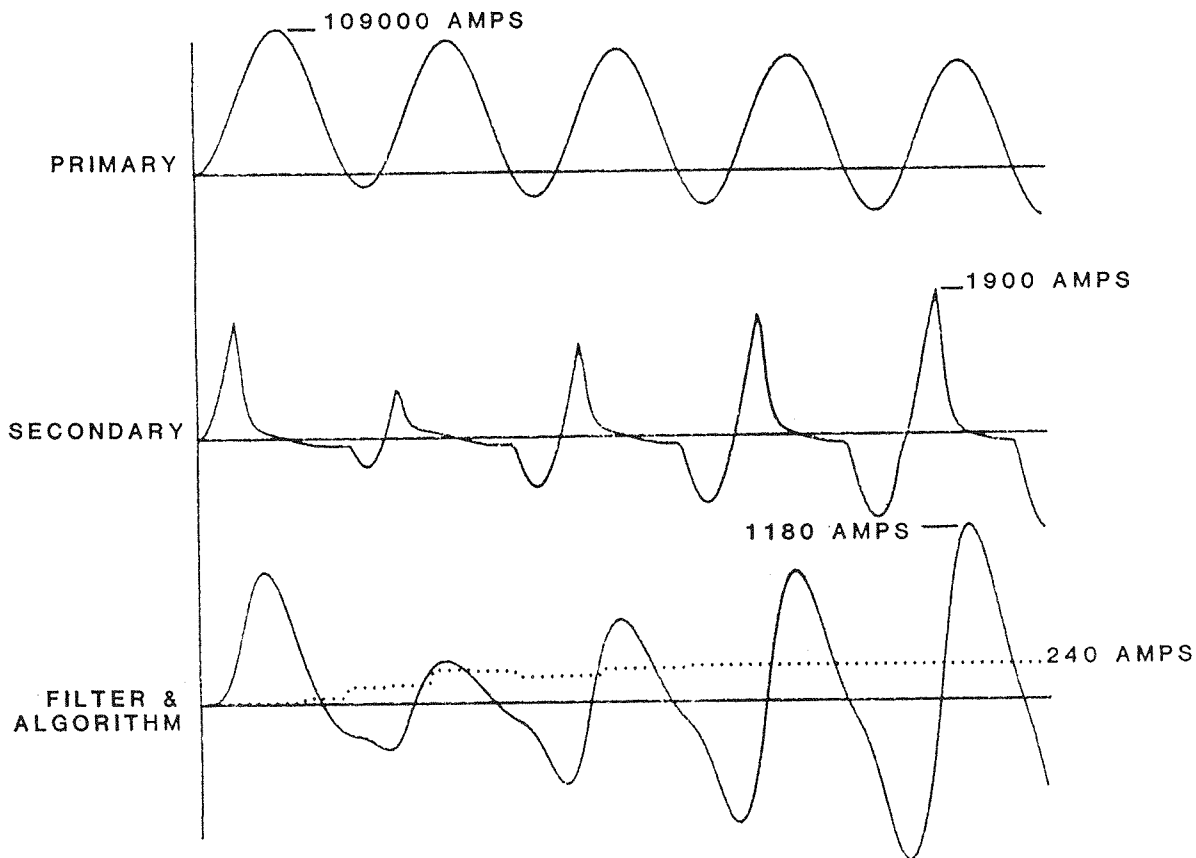


Figure 9. Computer simulation of 40,400 amp. test of a 150/5 ct showing primary and secondary currents. Lower trace shows filter response and output of sampling algorithm.

The oscillogram for the 150/5, 40,400 amp. case was shown above in Figure 1. to illustrate the severely distorted secondary waveform in a low ratio ct at 269 times its rating. Figure 9 is the computer simulation of this case in which the upper two traces are the primary and secondary currents. The lower trace shows the sine wave like response of the anti-aliasing filter which is sampled to provide the input for the digital filter. The superimposed plot is the magnitude of the digital output of the sampling algorithm.

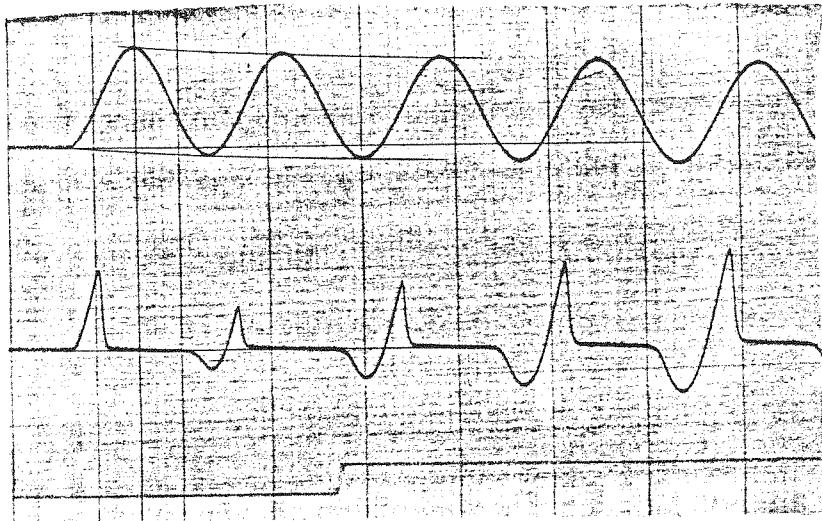


Figure 10 Oscilloscope of 10,000 amp. test showing primary and secondary current of a 50/5 ct. Lower trace shows Trip signal.

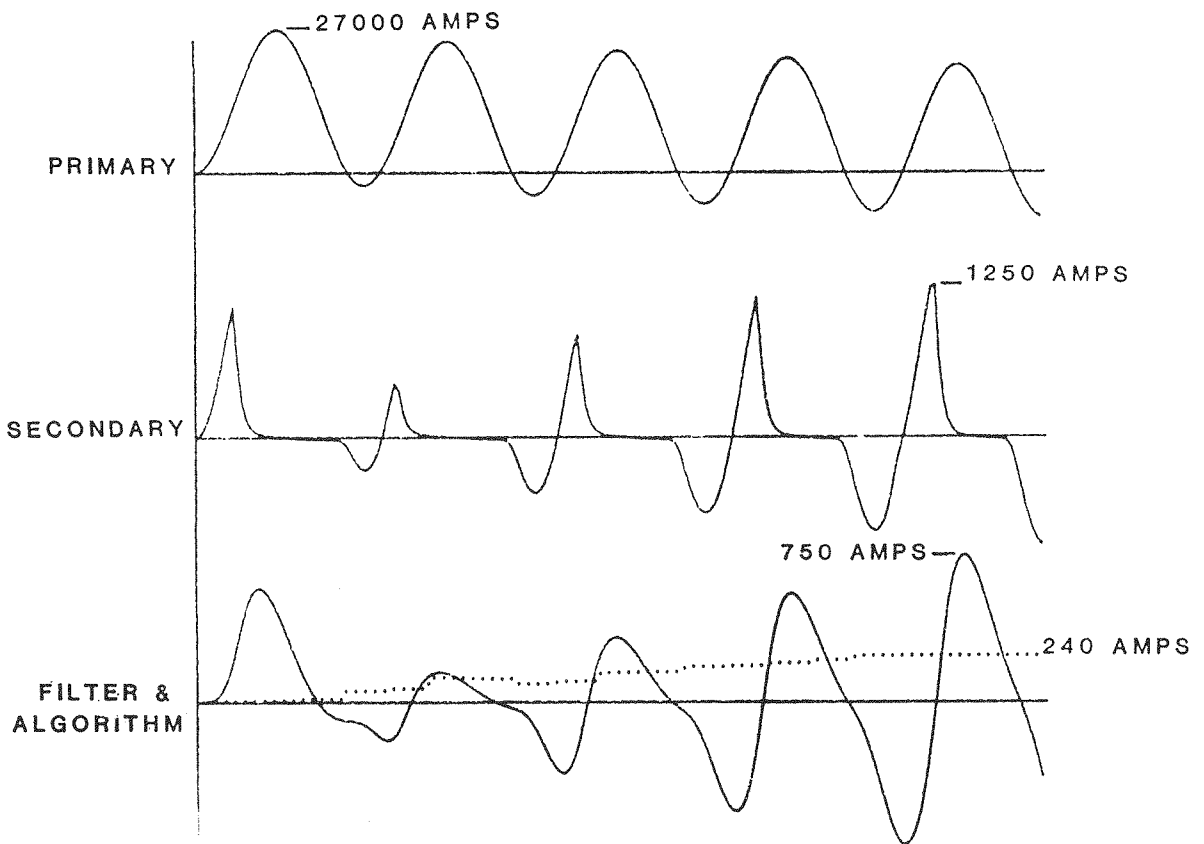


Figure 11. 10,000 amp simulation of 50/5 ct primary and secondary currents, filter response and output of sampling algorithm.

Comparison of the detail of the saturated secondary wave form with the actual oscillogram of Figure 1 verifies the accuracy of the simulation. Figure 10 is the oscillogram of the 50/5, 10,000 amp. case and Figure 11 shows the computer simulation of the same case for comparison. The plot shows that the simulation produces an accurate representation of the current waveform for the desired input condition.

## Conclusions

Selected high current tests using a range of cts and current magnitudes verified the operation of a digital overcurrent relay. The relay which employs a four sample/cycle digital filter tripped satisfactorially for all the test conditions.

The tests verified the accuracy of a detailed computer simulation which models the primary ct, the relay input transformer, anti-aliasing filter, PGA and relay sampling and magnitude estimation algorithm.

The computer simulation provides an economical means for verifying operation of a relay under any fault current condition allowed by switchgear standards. It is especially useful in testing relay operation with severely saturated cts due to extremely high fault currents.

The simulation allowed comprehensive testing of anti-aliasing filter response with impulse like input due to severely saturated cts. It is the sine-like response of the filter that is sampled to produce the maximum digital output for all currents beyond the linear range of the A/D converter

## References

1. ANSI C37.20 IEEE Standard for Switchgear Assemblies, p 18, Table 6.
2. Attwood, Stephen S., "Electric and Magnetic Fields", Dover Publications, Inc., New York, 1967 p 385.
3. Touhy, E.J. and Panek, J., "Chopping of Transformer Magnetizing Currents, part I: single phase Transformers", IEEE Transactions, Vol. PAS-97, No.1, Jan/Feb. 1978, pp261-268.
4. Samos, I., and Piccone, D.E., "The PS Method", Cardinal Printing Co. Sharon Hill, Pa.
5. Gill, William, "Easy to Use BASIC Program Analyzes Transient Response EDN, February 18, 1981, pp 33-136.