# RECENT USES OF SYMMETRICAL COMPONENTS IN RELAYS

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Presented to the
11th Annual
Western Protective Relay Conference
Spokane, Washington
October 23-25, 1984

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The introduction by C. L. Fortescue of the Method of Symmetrical Components in 1918 created little notice. His interest in the effect of single-phase railways on 3-phase power systems led him to the development of this extraordinary tool which, today, is a source of an essential part of our language. Power system engineers would be hampered for a way to describe with any precision, many of the basic phenomenon regarding, for example, rotating machinery, transformer connections, power system stability, and of course faults, without the words positive, negative and zero sequence.

In spite of the fact that it was a 1918 creation, symmetrical components, continues to be a vital analytical tool. Protective relaying has been one of the principal beneficiaries of Fortescue's work. Many time-honored relays would not exist in their present form if it were not for this tool. In this paper several modern uses will be described.

## CGRS

The CGRS relay was developed to provide a better means of detecting open and high resistance grounds on 4-wire distribution circuits. It is an electromechanical relay that uses a traditional zero sequence overcurrent unit to produce closing torque on an induction disc. Various restraint concepts were examined, each intended to change the effective level of zero sequence current at which tripping takes place, depending on some function of load current. Several had promise, but the one that was chosen has the utmost in simplicity, is a reasonably conventional electromechanical device and inherently, though not very obviously, produces restraining torque in proportion to $[I_1]^2 - [I_2]^2$. The restraint electromagnet consists of:

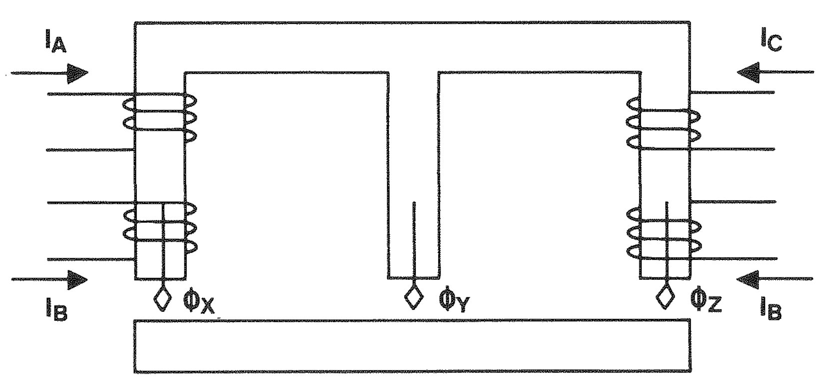


*Fig. 1. Restraint Electromagnet for CGRS Relay.*

The three leg fluxes are approximately:

$$\emptyset_X \propto (I_A - I_B) - (I_C - I_B)/2$$

$$\emptyset_Y \propto - (I_A - I_B)/2 - (I_C - I_B)/2$$

$$\emptyset_Z \propto (I_C - I_B) - (I_A - I_B)/2$$

Applying the method of symmetrical components to these fluxes:

$$3\emptyset_{X1} = \emptyset_X + a\emptyset_Y + a^2\emptyset_Z$$

$$3\emptyset_{X1} \propto (I_A - I_B) - (I_C - I_B)/2 + \frac{a}{2}[-(I_A - I_B) - (I_C - I_B)] + a^2 [(I_C - I_B) - (I_A - I_B)/2]$$

Collecting terms:

$$3\emptyset_{X1} \propto I_A \left(1 - \frac{a}{2} - \frac{a^2}{2}\right) + I_B \left(- 1 + \frac{1}{2} + a - a^2 + \frac{a^2}{2}\right) + I_C \left(- \frac{1}{2} - \frac{a}{2} + a^2\right)$$

$$3\emptyset_{X1} \propto \frac{3}{2} (I_A + aI_B + a^2I_C)$$

Then changing the constant of proportionality:

$$\emptyset_{X1} \propto \frac{1}{3} (I_A + aI_B + a^2I_C) = I_{A1}$$

Similarly:

$$\emptyset_{X2} \propto I_{A2}$$

$$\emptyset_{X0} = 0$$

Since restraining torque on this unit is proportional to approximately the magnitude of $\emptyset_{X1}$ squared minus the magnitude of $\emptyset_{X2}$ squared then it is proportional to $[I_{A1}]^2 - [I_{A2}]^2$.

This overall concept allows complete cancellation of restraint on line-to-ground faults on radial circuits where maximum sensitivity is desired for tripping and produces substantial restraining torque on load when the maximum residual load unbalance may exist and tripping is not desired.

It also allows sensitive detection of open conductors on 4-wire distribution circuits.

## LQS

Another recent development in relaying, requiring the use of symmetrical components is the LQS. Its principal function is to identify the faulted phase for single-pole tripping on line-to-ground faults. It is an inherent part of the LDAR pilot relaying system.

One could intuitively sense that a phase, faulted to ground could be identified by examining the phase relationship between zero sequence current and negative sequence current because the symmetrical component networks are placed in series to represent, for example, a "phase A"-to-ground fault. "A-phase" positive, "A phase" negative, and "A-phase" zero sequence currents are all in phase. For other phase-to-ground faults, the faulted phase negative sequence current in the fault is in phase with the zero sequence current. This relationship is shown in Figure 2.

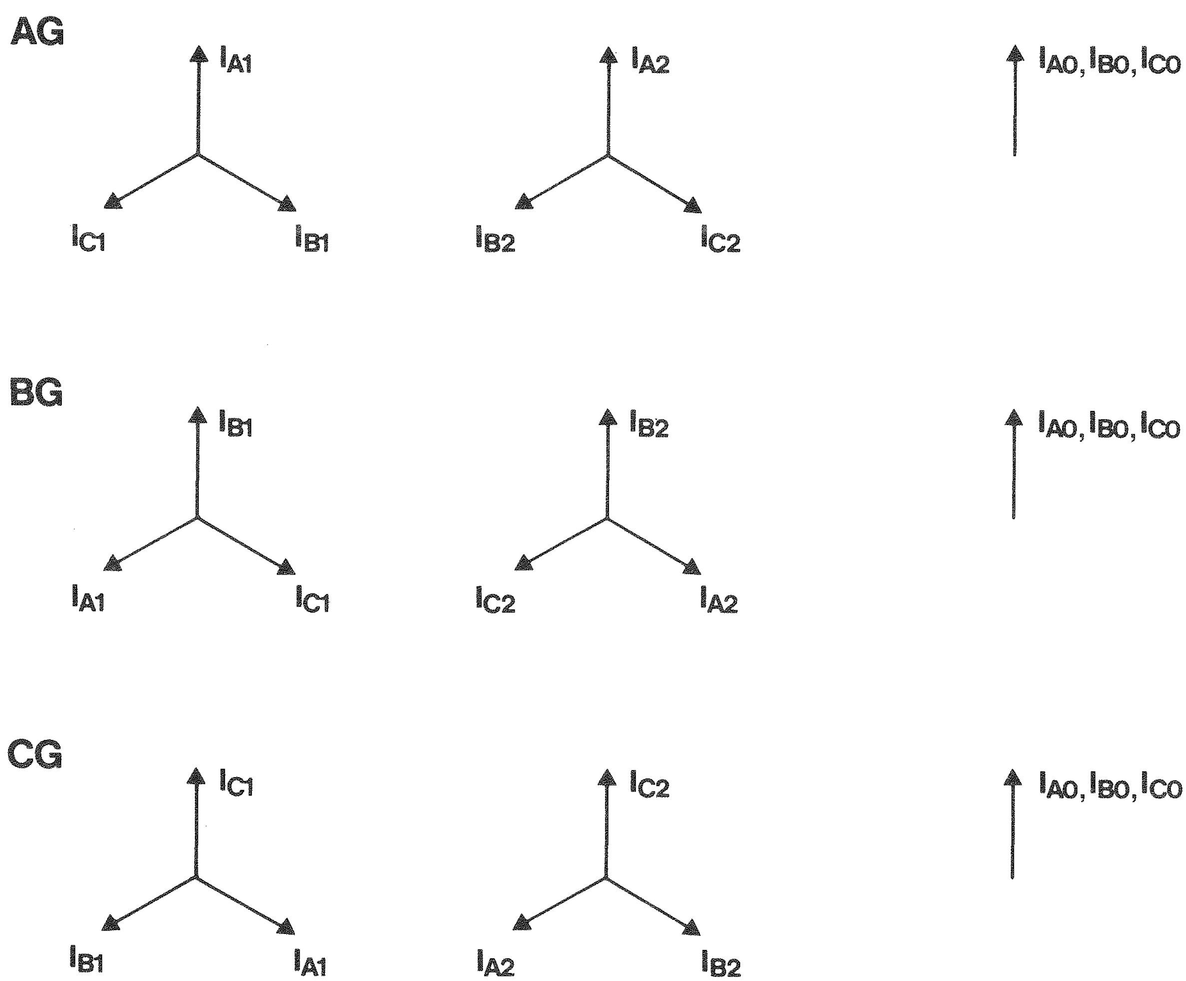


*Fig. 2. Relationships Between Phase Components at the Fault for Different Phase-To-Ground Faults.*

It is tempting to use solid state logic comparing only the phase relationship of zero sequence with negative sequence current to choose the faulted phase. This would be disappointing because, for example, a B-C-G fault causes zero sequence current to be in-phase with A∅ negative sequence current, assuming equal phase angles for the zero and negative sequence networks. Figure 3 shows how this would come about. This would select A∅ for single pole tripping for the B-C-G fault, an obviously unsatisfactory result.

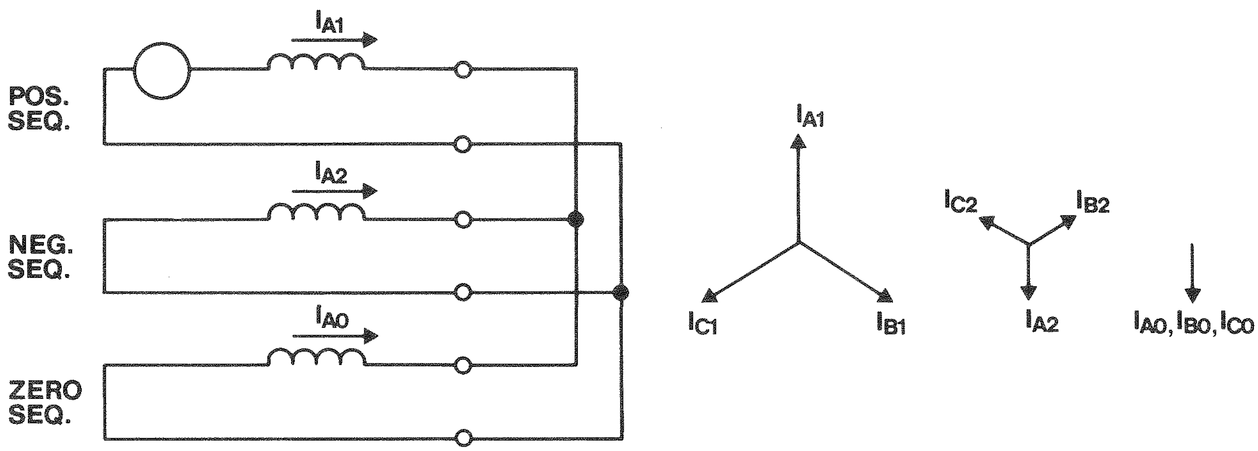


*Fig. 3. Relationship Between Phase Components at the Fault for A B-C-Ground Fault.*

To overcome this problem, logic is added to examine the angular relationship between zero sequence current, negative sequence current for each phase and and cross-polarizing voltage. For example, Figure 4 shows the logic for "A phase" selection. Note that positive coincidence (the time during which all three quantities are simultaneously positive) is achieved for a full 8.33 ms (180 degrees) for the in-phase condition shown for the A-phase-to-ground fault case. No coincidence exists for the phase relationships shown for the B-C-ground case. In a practical case, these exact relationships would not be realized, but at least 5.5 ms (120 degrees) coincidence would be expected for the phase-to-ground fault.

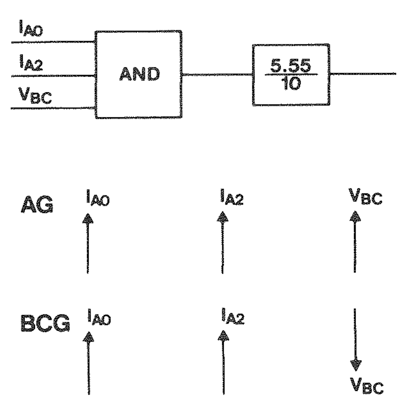


*Fig. 4. Conceptual Logic for Identifying Phase-To-Ground Faults with LQS Module.*

Similar comparisons for the other two phases would allow B or C phase to be selected for single pole tripping for B-to-ground or C-to-ground faults respectively.

## LCB

Another significant application for the use of symmetrical components is in the LCB pilot relay. In this relay it is necessary to generate single phase quantities proportional to each of the three components - positive, negative and zero, to allow their individual weighting in the equation:

$$V_f \propto C_1 I_{A1} + C_2 I_{A2} + C_0 I_{A0}$$

where $V_f$ is the single phase sinusoidal quantity generated at each end of a transmission line by this filter circuit for transmission over the fiber-optic or tone channel for comparison with the similar quantity at the other end of the transmission line.

Many ways of implementing this equation exist. Care must be exercised to preserve the phase angle relationship and magnitude between sequence quantities. Figure 5 shows a partial schematic of the method by which this was accomplished in the LCB pilot relay.

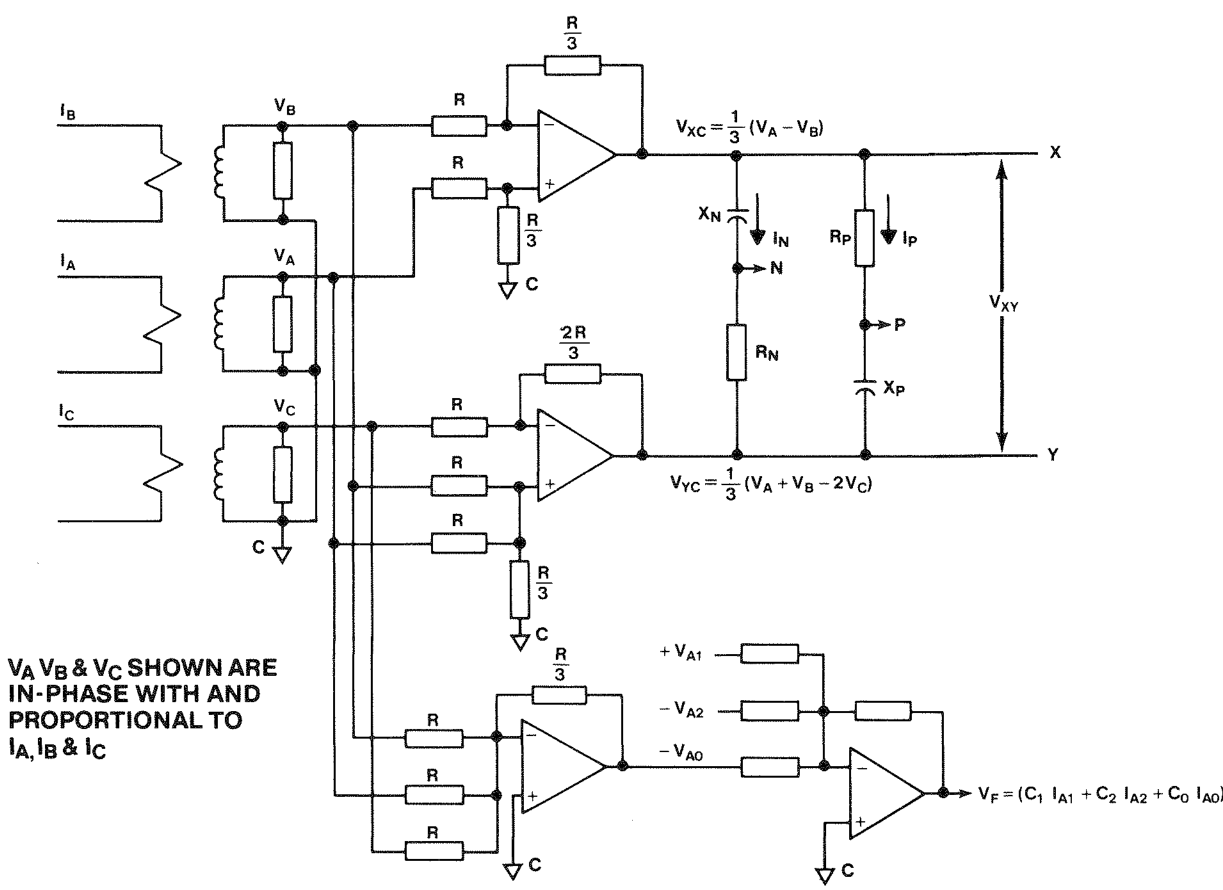


*Fig. 5. LCB Pilot Relay Sequence Filter.*

Figure 6 describes how the two independent output circuits produce voltages that are proportional to the positive and negative sequence components respectively of the input currents. The voltage drop from P to C (common) is proportional to the positive sequence component of the AØ current and from N to C is proportional to the negative sequence component of the AØ current.

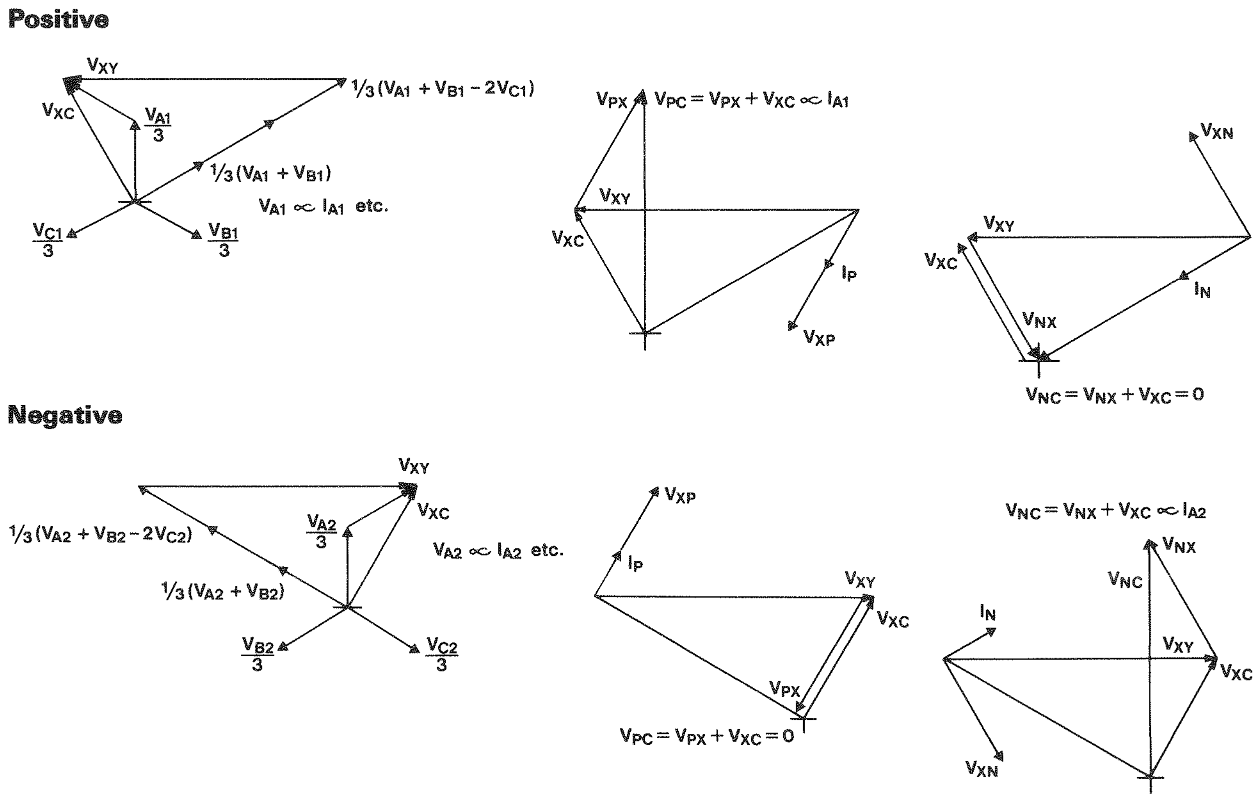


*Fig. 6. Outputs of LCB Filter with Positive and Negative Sequence Current Inputs.*

This can be shown analytically by:

$$V_{PC} = V_{XC} - I_P R_P$$

$$3V_{PC} = (V_A - V_B) - \left[\frac{(V_A - V_B) - (V_A + V_B - 2V_C)}{R_P - jX_P}\right] R_P$$

$$3V_{PC} = (V_A - V_B) - [\ V_A - V_B - V_A - V_B + 2V_C\ ]\ \tfrac{1}{2}\ \underline{/+60}$$

$$3V_{PC} = V_A - V_B + V_B\ \underline{/60} - V_C\ \underline{/60}$$

$$3V_{PC} = V_A + V_B\ (1\ \underline{/60} - 1) - V_C\ \underline{/60}$$

$$3V_{PC} = 1 + aV_B + a^2V_C \equiv 3V_{A1}$$

$$V_{PC} = V_{A1}$$

Similarly:

$$V_{NC} = V_{XC} - I_N(-jX_{CN})$$

$$3V_{NC} = V_A - V_B + \left[\frac{(V_A - V_B) - (V_A + V_B - 2V_C)}{R_N - jX_{CN}}\right] jX_{CN}$$

$$3V_{NC} = (V_A - V_B) + [V_A - V_B - V_A - V_B + 2V_C]\,\frac{1}{2}\,\underline{/120}$$

$$3V_{NC} = V_A - V_B(1\,\underline{/120} + 1) + V_C\,\underline{/120}$$

$$3V_{NC} = V_A + a^2V_B + aV_C \equiv 3V_{A2}$$

$$V_{NC} = V_{A2}$$

**LRQ/LRQ-2**

Negative sequence directional overcurrent relaying has distinct advantages over zero sequence directional relaying where mutual effects are present or on long line applications where a low zero sequence source impedance exists. This scheme requires a phase angle comparison between $V_{A2}$, the negative sequence voltage at the relay location and $I_{A2}$ the negative sequence current delivered to the relay by the ct's. Figure 7 shows the relationship between these two quantities.

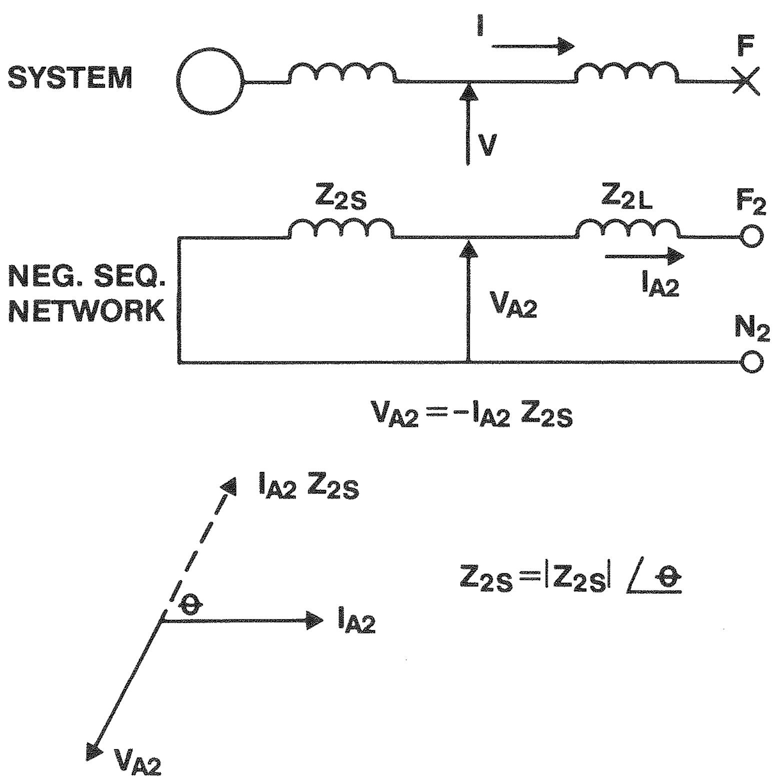


*Fig. 7. Schematic of Negative Sequence Network and Phasor Diagram.*

Figure 8 shows the phasor relationship used to extract negative sequence voltage from a set of three input voltages having any degree of unbalance. $V_{AB} + V_{BC} \underline{/-60}$ will always equal zero when positive sequence voltage is applied as Figure 8a shows. It will always equal zero when zero sequence voltage is applied because phase-to-phase voltage contains no zero sequence component, ever. $V_{AB}$ and $V_{BC}$, then, are always zero if only zero sequence voltage is present. This leaves negative sequence voltage as being the sole contributor to any remainder in $V_{AB} + V_{BC} \underline{/-60}$.

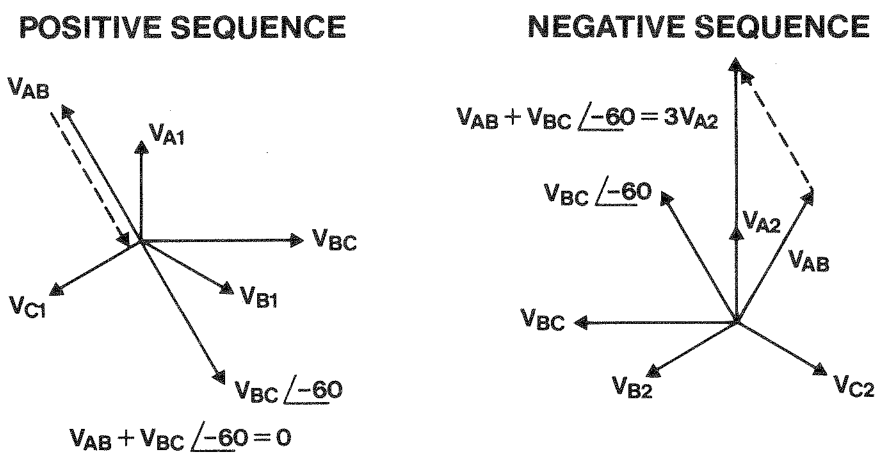


*Fig. 8. Phasors for Positive and Negative Sequence Voltage Segregation.*

To check this we observe:

$V_{AB} = V_{AG} - V_{BG}$

$V_{BC} = V_{BG} - V_{CG}$

Then $V_{AB} + V_{BC} \underline{/-60} = V_{AG} - V_{BG} + (V_{BG} - V_{CG}) \underline{/-60}$

This can be rearranged to:

$V_{AG} + V_{BG} (1 \underline{/-60} -1) - V_{CG} \underline{/-60}$

Which becomes:

$V_{AG} + V_{BG} \underline{/240} + V_{CG} \underline{/120}$, which we recognize as $3V_{A2}$

Thus $V_{AB} + V_{BC} \underline{/-60} = 3V_{A2}$

Figure 8b shows this pictorially.

By using input transformers similar to those in Figure 5 to develop voltages proportional to, and in-phase with, the phase currents, and then manipulating these voltages with summing and phase shifting operational amplifiers to produce results similar to that described in Figure 8, a quantity proportional to and in-phase with negative sequence current (or more precisely proportional to and in-phase with the negative sequence component of A-phase current) can be developed for angular comparison with

$V_{A2}$, developed in a similar way. This simple comparison described in Figure 9 provides a directional unit that is oblivious to zero sequence mutuals and is more compatible with the levels of available relaying quantities for long line far-end fault sensing. Negative sequence mutual effects are, in any practical sense, small enough to ignore.

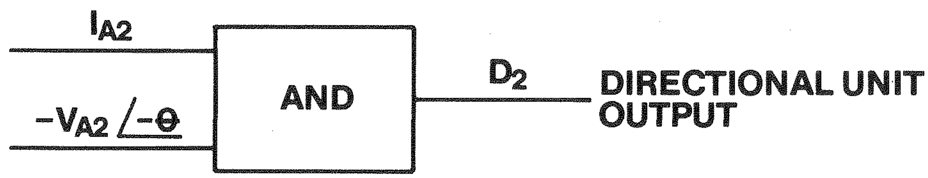


*Fig. 9. Fundamental Logic for $V_2$ $I_2$ Directional Unit.*

Both the LRQ and LRQ-2 Uniflex modules use negative sequence current and voltage for pilot directional sensing, each providing improvement in the presence of mutual effects over a zero sequence directional unit. The LRQ-2 is preferred over the LRQ for long line applications because of its use of a negative sequence current fault detector as compared to the LRQ zero sequence fault detector.

## Conclusion

The passage of time has amplified the importance of Symmetrical Components as a relaying tool. It has endured 66 years of extreme variations in relay design practices and remains as an irreplaceable analytical tool and fortifier of sound thought process to relate the power system to realistic protective relaying systems.

## Appendix I

The method of symmetrical components allows representation of unbalanced conditions on an n phase system with n sets of balanced conditions. They are identified for a three-phase power system as positive, negative and zero sequence quantities.

For example:

Positive sequence phasors for the three phases are equal in magnitude, 120 degrees apart and have normal phase sequence. Negative sequence is similar except there is opposite-to-normal phase sequence. Zero sequence quantities are equal in magnitude and in-phase. All sequence phasors rotate in a counter-clockwise direction. The magnitudes and phase angles of the components may theoretically have any relationship to one another.

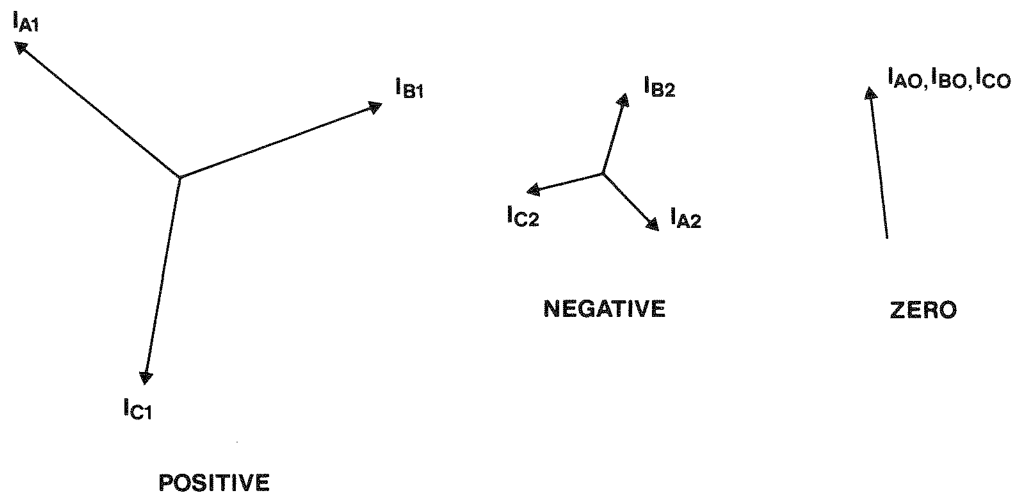


*Fig. I-1. Typical Symmetrical Components.*

Adding together the individual phase quantities from Figure I-1, we get:

$I_A = I_{A1} + I_{A2} + I_{A0}$

$I_B = I_{B1} + I_{B2} + I_{B0}$

$I_C = I_{C1} + I_{C2} + I_{C0}$

From the equations above we can also write:

$I_A = I_{A1} + I_{A2} + I_{A0}$

$I_B = a^2 I_{A1} + aI_{A2} + I_{A0}$

$I_C = aI_{A1} + a^2 I_{A2} + I_{A0}$

Where a is the operator $1 \underline{/120^\circ}$

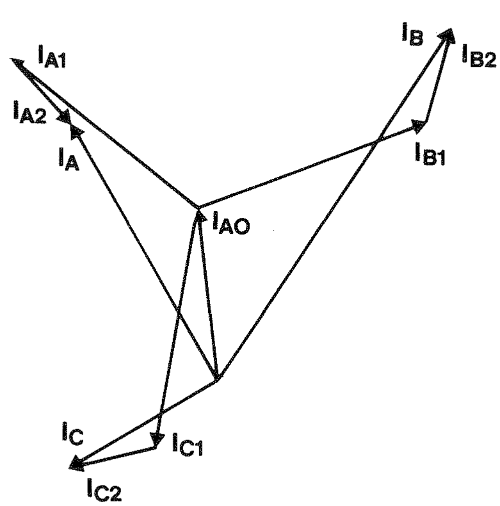


*Fig. I-2. Combination of Components to Produce Phase Currents.*

Solving these equations for $I_{A1}$, $I_{A2}$ and $I_{A0}$, we get:

$$I_{A1} = \frac{1}{3}(I_A + aI_B + a^2 I_C)$$

$$I_{A2} = \frac{1}{3}(I_A + a^2 I_B + aI_C)$$

$$I_{A0} = \frac{1}{3}(I_A + I_B + I_C)$$

Performing these operations on the phase currents of Figure I-2, it is seen in Figure I-3 that the sequence quantities can be developed from the phase values as well as vice-versa.

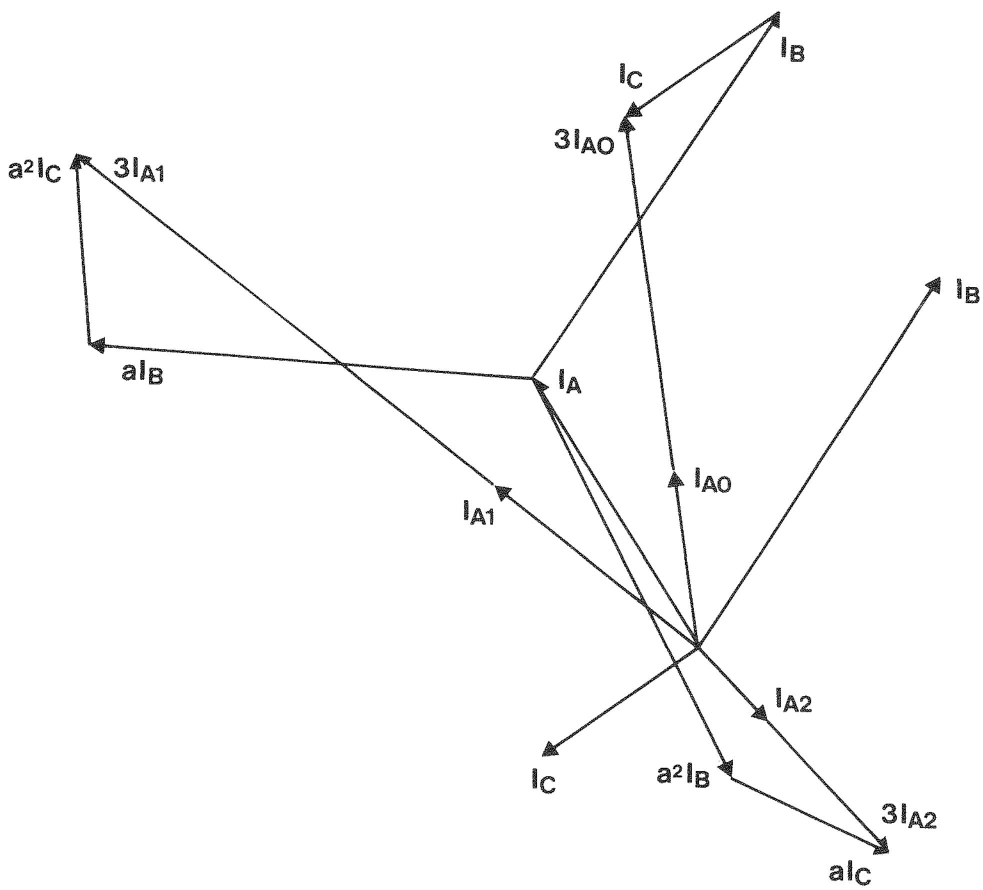


*Fig. I-3. Extraction of Symmetrical Components from Phase Currents.*

While unbalanced currents can be manipulated into sets of balanced currents, unbalanced voltages may be handled similarly. Balanced phase currents - positive, negative or zero sequence flowing through balanced impedances in the three phases, produce voltage drops of only the same sequence. For example, negative sequence currents, in flowing through balanced impedances, produce only negative sequence voltage drops.

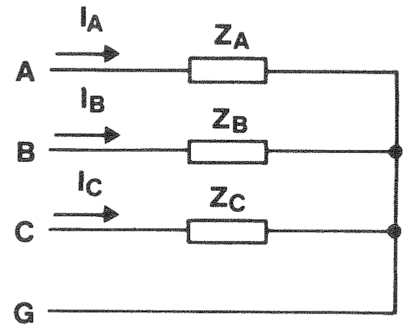


*Fig. I-4. Voltage Drops in Phase Impedances.*

$$V_{AG} = I_A Z_A = (I_{A1} + I_{A2} + I_{A0}) Z_A$$

$$V_{BG} = I_B Z_B = (a^2 I_{A1} + a I_{A2} + I_{A0}) Z_B$$

$$V_{CG} = I_C Z_C = (a I_{A1} + a^2 I_{A2} + I_{A0}) Z_C$$

$$V_{A1} \equiv \frac{1}{3} (V_{AG} + a V_{BG} + a^2 V_{CG})$$

$$V_{A1} \equiv \frac{1}{3} [(I_{A1} + I_{A2} + I_{A0}) Z_A + a(a^2 I_{A1} + a I_{A2} + I_{A0}) Z_B + a^2 (a I_{A1} + a^2 I_{A2} + I_{A0}) Z_C]$$

Collecting:

$$V_{A1} = \frac{1}{3} [I_{A1} (Z_A + a^3 Z_B + a^3 Z_C) + I_{A2} (Z_A + a^2 Z_B + a Z_C) + I_{A0} (Z_A + a Z_B + a^2 Z_C)]$$

for $Z_A = Z_B = Z_C$, since $a^3 = 1$ and $1 + a + a^2 = 0$

$$V_{A1} = I_{A1} Z_A$$

Similarly $V_{A2} = I_{A2} Z_A$ and $V_{A0} = I_{A0} Z_A$

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