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PRIMER ON DISTANCE RELAYS

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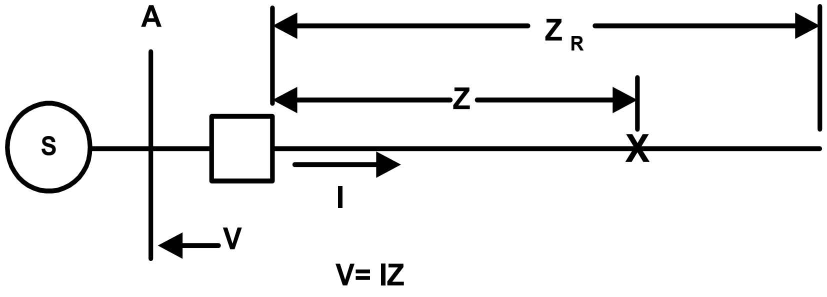
Walter A. Elmore

Introduction

Over the years, a wide variety of methods has been devised to identify the location of a fault on a transmission line. They vary from the simplest conceivable arrangement to highly complex mathematical entities. This paper reveals no new methods and is primarily intended to describe in the simplest terms the manner in which different technologies have been applied to perform the basic function of differentiating between faults for which operation is desired and those for which it is not. Most of the relays that are applied to recognize the difference are not truly "distance" relays, in that, they do not measure the distance to a fault. They would be more accurately called "impedance" relays because they all respond to the ratio of voltage to current. However, this author would not have the temerity to suggest any change in the terminology that has evolved historically.

Fundamental Premise

Figure 1 describes the basic premise for distance relaying. A fault at X will cause current flow to the fault from the source, S. This current produces a voltage drop, IZ, in the impedance to the fault. At substation A, access is available to both this voltage and to the current producing this voltage. Comparison of the voltage to the current allows a determination to be made as to whether or not the fault is within the distance for which tripping is desired. If the fault is at any location within the desired "reach" of the relay, operation takes place. A distance relay does not identify the distance to the fault, only whether it is within or beyond the borderline for which the relay is set.



APPARENT Z AT RELAY = V / I = IZ / I = Z
IF SETTING Z_R > Z , RELAY OPERATES

Figure 1 Basic Premise for Distance Relaying

Quantities Available

Clearly, the voltage and current to be compared at the relay location for a given type of fault must be chosen carefully. An incorrect choice will lead to an incorrect decision, producing, generally, failure to trip for a case for which tripping is desired. Most relays have all fault combinations covered, though, in discrete form, phase and ground relays perform their functions independently. The comparisons identifying the lowest impedance are allowed to dictate the tripping decision.

Figure 2 describes the nature of the available information for two important phase fault types.

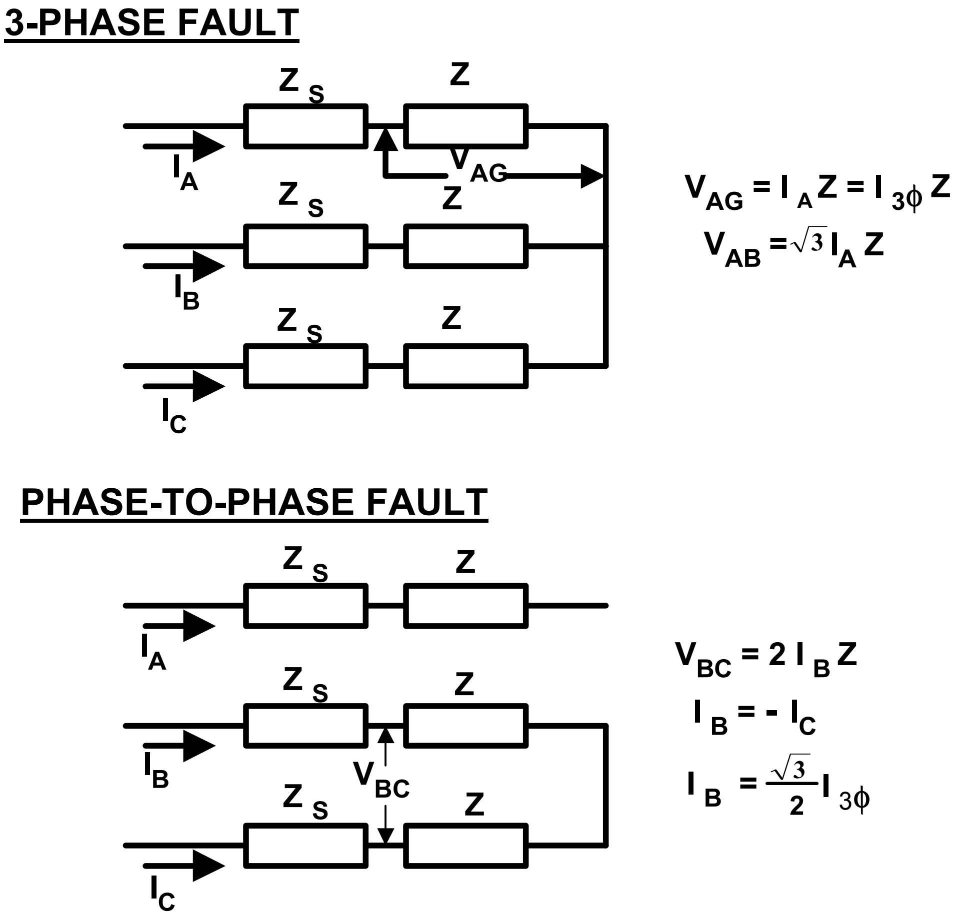


Figure 2 Contrast of Quantities for Different Phase Faults

Figure 2 is intended to represent a system having a source impedance of Z_S, and an impedance from the relaying location to the fault, Z. The voltages for the two phase-fault cases are quite different, and an appropriate choice must be made for comparison with the currents to assure a proper identification of impedance to the fault.

Table 1 explores the possibilities that are available for comparison during phase faults. Using wye voltage (phase-to-ground) and wye current would provide a suitable measurement for three-phase faults, but a terrible measurement for phase-to-phase faults. The wye voltage contains a component that is not related to the impedance from the relay to the fault. For this phase-to-phase fault, with zero voltage between the faulted phases, there is not at the fault, zero voltage to ground.

Using delta voltage (phase-to-phase voltage) and wye current, there is some improvement, but there is still a 15% difference in the apparent impedance to a phase fault depending on which type of phase fault is involved.

Impedance measurement using delta voltage and delta current provides identical definition of fault location for the two cases.

Table 1

Measuring Quantities	3ϕ	$\phi\phi$	**Comment**
$\frac{V_Y}{I_Y}$	$\frac{IZ}{I} = Z$	$\frac{IZ - j0.5V'}{I} \neq Z$	**N.G.**
$\frac{V_\Delta}{I_Y}$	$\frac{\sqrt{3}}{I}\frac{IZ}{} = \sqrt{3}Z$	$\frac{2IZ}{I} = 2Z$	**N.G.**
$\frac{V_\Delta}{I_\Delta}$	$\frac{\sqrt{3}}{\sqrt{3}}\frac{IZ}{I} = Z$	$\frac{2IZ}{2I} = Z$	**O.K.**

N.G. Not suitable for both 3ϕ and $\phi\phi$
V_Δ = Phase-to-phase voltage (V_A-V_B etc)
I_Y = Phase current (I_A etc.)
I_Δ = Delta current (IA - IB etc.)

Virtually all phase distance relays acknowledge, in one way or another, the requirement to use delta quantities . Through this expedient, both three-phase and phase-to-phase faults can be located through a common measurement in each of the phases.

Ground Relays

Ground relays require more elaborate analysis. For an A-phase-to-ground fault at the location shown in figure 1, the voltage to ground at the relay will be $I_a Z_{1L} + I_0 (Z_{OL} - Z_{1L})$, because of the influence of the zero-sequence current return path. By using A-phase voltage-to-ground and comparing it to $\{I_a + I_0 [(Z_{OL} - Z_{1L}) / Z_{1L}]\} Z_C$, the fault can be located as being within or outside of the desired reach of the relay. Similar comparisons for the other phases lead to an identification of the location of ground faults on those phases. Using lead-lag relationships much like those that will be described later for phase distance relays, ground distance relaying is accomplished in a very straightforward way. This paper makes no effort to describe the influence of mutual effects or of ground fault resistance.

Phase Relays

Implementation of the phase relaying function has taken many intriguing forms. One of the earliest distance relays had an operating time inversely proportional to the apparent impedance applied to the relay. To accomplish this, a balanced-beam and an induction disc unit were used. Above the induction-disc overcurrent unit pickup level, the disc turned. As it did, a spring was wound up through a gear-train. The spring, then, worked on the balanced beam against the torque produced by the voltage coil. The higher the voltage, the farther the induction disc had to travel to produce enough spring-torque to close the contacts. Needless to say, a Swiss watchmaker's skills were needed to design and build such an intricate device, but indeed, an adequate tribute is hardly possible for the men who conceived it. It worked and worked well, but we know what happened to the Swiss watch business.

Phase distance relays gravitated to a less intricate design which is represented graphically in figure 3. This simple mechanism made a distinct impact on the nature of distance relaying.

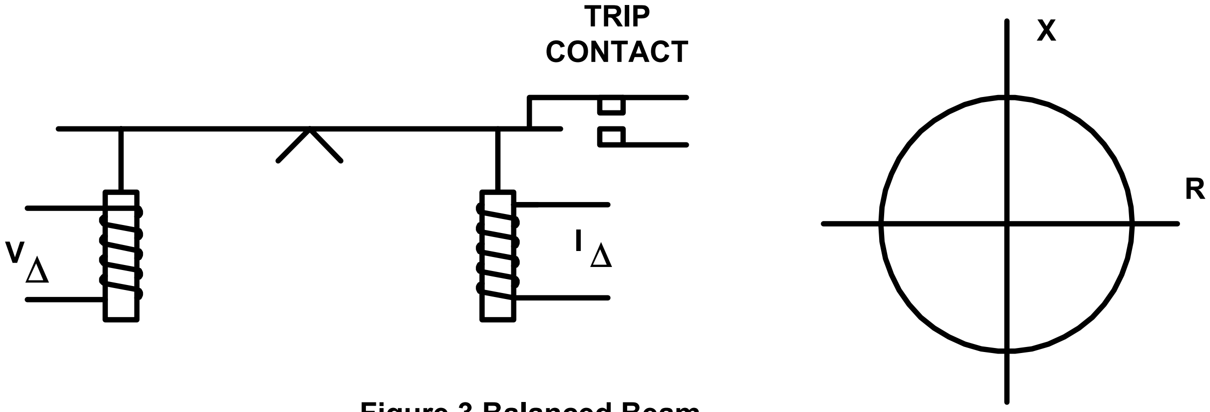


Figure 3 Balanced Beam

No faster overall trip time has been realized than with this relay, but after approximately 15 years of stellar service, other considerations became evident. Line length and loadability began to take their toll. The characteristic operating circle of this relay is shown at the right in figure 3 with the operating area being inside the circle. It is apparent that this single-phase unit is oblivious to phase angle and, of course, requires the support of a directional unit for each phase. Since it's reach is the same at load angles as at fault angles, distinction must be made by assuring that the reach is short enough to **exclude** load ohms.

As lines became longer and more heavily loaded, the inadequacy of this unit forced the creation of the modification shown in figure 4. This design introduced a voltage proportional to current into the voltage circuit. That voltage introduced was forced to lead the current by the natural power-factor angle of the transmission line. With this current-dependent voltage subtracted from the voltage at the relay terminals, the relay became highly sensitive to lagging fault current and relatively insensitive to currents in the neighborhood of unity power-factor. It still required a separate directional unit to screen those reverse faults whose apparent ohms fell within the modified circle.

The characteristic circle of figure 4 is shown with vectors that represent the minimum fault condition and the maximum load condition. Operation, of course, occurs when the point of the vector is inside the circle.

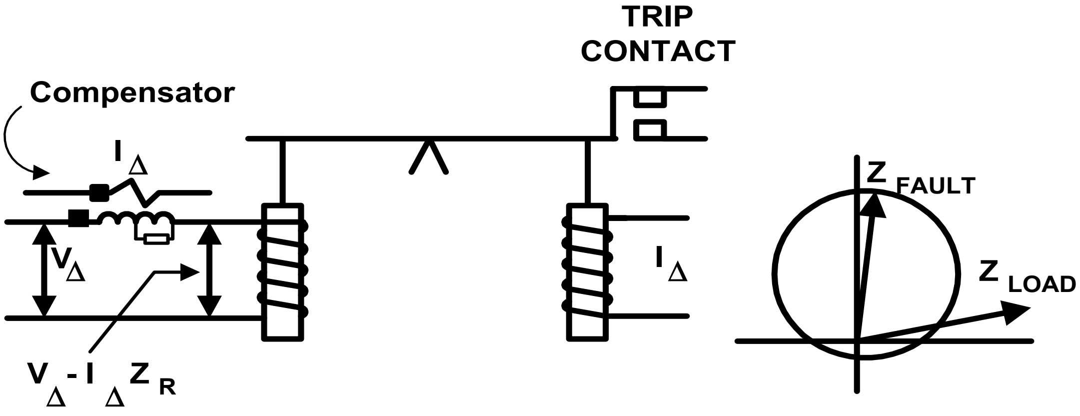


Figure 4 Offset Impedance Relay

The torque relationships are similar to those experienced with figure 3. The phase-to-phase voltage, $V\Delta$ (V_{AB}, for example) modified by the current, produces a pull on the left side of the balanced beam that is proportional to the square of that voltage, and the delta current (I_A - I_B , for example) produces a pull on the right side that is proportional to the square of the current. When the pull on the two sides are equal, the relay is in balance. The relay is in balance when the setting is satisfied. With a reduction in voltage to, say, 50% of that at balance, and a reduction in current to 50% of that at balance, the relay **remains in balance.** This is probably the source of the term " balance- point." Any increase in the current while holding the input voltage constant will cause the relay to operate. Any decrease in the input voltage, while holding the current constant will cause the relay to operate. In other words, the relay is responsive to the apparent impedance associated with the quantities at its terminals.

Cylinder Unit

Another extremely useful element made its entrance around 1958 . This is the cylinder unit. By proper manipulation of the inputs, and design of coils, this element can be a directional unit, an undervoltage unit, an overcurrent unit, and for our consideration here, an impedance unit.
The idea of the cylinder unit draws heavily from 2-phase motor technology. The forward torque is proportional to the area of the applied voltage triangle, irrespective of the degree of unbalance it may contain. Reverse phase sequence voltage applied to the cylinder unit will cause it to rotate in the reverse-to-normal direction. Rotation is prohibited except for a very small angle by a "stop" in one direction and the stationary contact in the other. Figure 5 describes one implementation of a distance unit for phase fault recognition. For this relay, the behavior is defined in terms of the XYZ phase sequence or the XZY phase sequence. With normal voltage applied the phase sequence is XYZ .By unique manipulation of the input quantities, a fault within the reach of the relay and in the proper direction produces an XZY phase sequence. We shall see how this is accomplished.

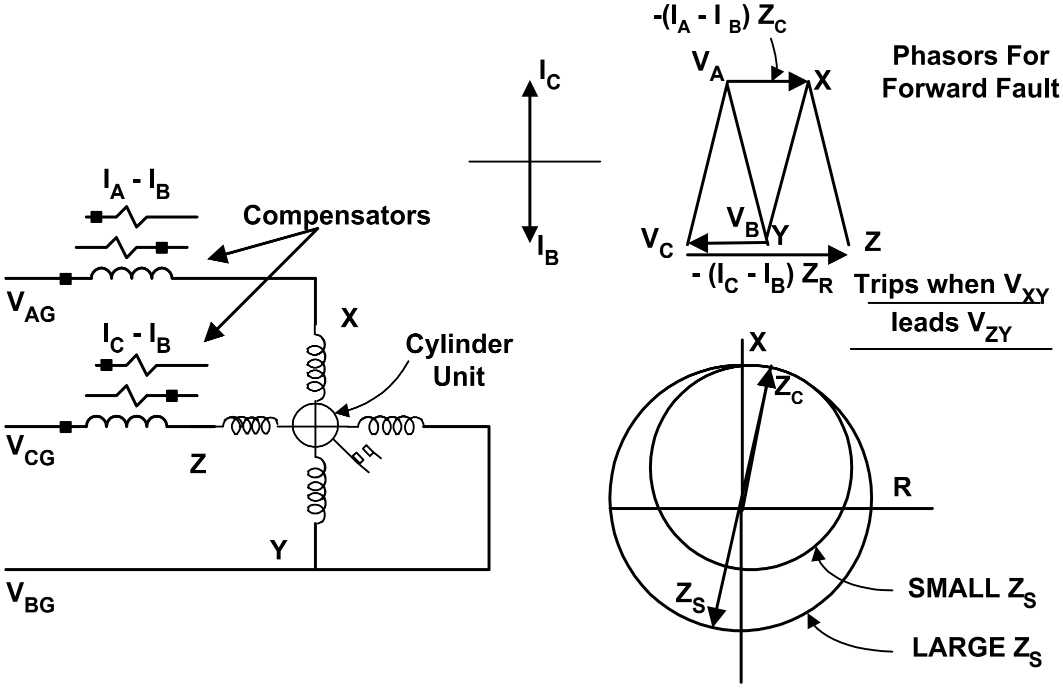


Figure 5 Cylinder Unit Distance Relay

The unit described in figure 5 is called a phase-to-phase unit. Its primary function is to detect all phase-to-phase and, in conjunction with a "three-phase" unit, all phase-to-phase-to-ground faults. Its unique quality is that this single element is able to detect all combinations of such faults. Note that the critical measurement for the B-to-C phase fault shown here is between the delta voltage, $V_{CB,}$ and the voltage (Ic - Ib) Z_C **.** Its behavior is such that it will operate when the negative sequence voltage (of the XYZ voltage triangle) exceeds the positive sequence voltage. Since a 3-phase fault contains no (or little, if the phase impedances are unequal) negative sequence voltage, the relay will not operate for such a fault. Similarly, **it will not operate for load or power swings and therefore need not be blocked against operation for such.**

Figure 5 shows that the characteristic circle of the relay. Its reach is fixed at the forward point by the setting of the relay, and is defined in the opposite direction by the source impedance. This should be interpreted with care because the region to the left of the vertical line is a negative resistance area, **not a reverse direction area.** The circle definition and reverse direction faults are two totally different things. The circle for reverse direction faults lies totally in the third quadrant. The relaying unit is **highly** directional, and unlike the relays of figures 3 and 4, requires no assistance from a directional unit to identify the direction to a fault nor a blinder unit to differentiate between a fault and a power swing. It also requires no "memory action" to provide a reference sense for a zero-voltage phase-to-phase fault. Finally, the compensator shown in figure 5 is a current to voltage transformer that has an air gap in the core, making the unit virtually impervious to influence from the dc component in the fault current. It, further, is adaptive so that it automatically adjusts the operating circle location to accommodate the effect of fault resistance with out-of phase sources (load flow). This unit has extraordinary credentials in terms of such unique qualities as well as successful performance in the field.

Obviously, using this concept, a different kind of unit is required to identify and locate 3-phase faults. A separate cylinder unit is used having a single "A-phase" compensator. The current generates a voltage (proportional to the ohmic setting) that is subtracted from the "A-phase" voltage establishing point X in the XYZ triangle. Points Y and Z in the triangle are fixed by V_{CB}. As with the other cylinder distance unit, operation takes place when V_{XY} leads V_{ZY}. With the balanced 3-phase fault, only a single impedance measurement is required. Its characteristic is a "mho" circle, that has a fixed forward reach and goes through the origin. To complement the phase-to-phase unit for $\phi\phi$G faults, the compensator actually uses I_A - $3I_0$ (or in special cases I_A - I_0) to generate the compensator voltage.

Contrasted with the phase-to-phase unit of figure 5, the 3-phase unit is dependent on "memory action" to trip for zero-voltage faults. Closing into a bolted 3-phase fault with line-side potential transformers causes this 3-phase unit to fail to operate. Other elements must be included in the relaying system, to assure operation in this case, such as, "close-into-fault logic."

The behavior of these compensator distance relays can be examined in several ways. The following questions may be asked:

- Is the phase sequence of the voltage at the cylinder unit XZY?
- Does the V_{X2} voltage at the cylinder unit exceed the V_{X1}?
- Does voltage V_{XY} lead V_{ZY}?

If the answer to any of these questions is yes, the relay will close its contacts if the torque produced overcomes the small spring restraint.

Other Examples

Table 2 shows that the concept of an "operating" phasor and a "polarizing" phasor can be applied to create a variety of important distance relaying functions. Only a few are shown.

TABLE 2

FUNCTION	OPERATING	POLARIZING
ΦΦ Distance	$V_{AB} - I_{AB} Z_C$	$V_{CB} - I_{CB} Z_C$
3Φ Distance	$V_{AB} - I_{AB} Z_C$	$j V_{AB}$
3Φ Distance	$V_{AB} - 1.5 (I_A - 3I_0)Z_C$	V_{CB}
Loss of Field	$V_{AN} + I_A Z_A$	$j (V_{AN} \pm I_A Z_C)$
Out-of-Step	$V_{AB} - I_{AB}Z_F$	$j (V_{AB} + I_{AB}Z_R)$
GND Reactance	$I_A + k I_0$	$V_{AG} - (I_A + k I_0)X_C$
GND Distance	$V_{AG} - (I_A + k I_0)Z_C$	V_{CB}

Except for the ΦΦ unit, all of the distance units described in the table are single phase
For the fault sensing relays, similar units are required for the other two phases
V_{CB} is, for example, voltage drop from C phase to B phase
All of the Z's are distance settings and contain an angle

Solid State Distance Relays

V_{XY} leading V_{ZY} served as an excellent operating criterion for the transistor version of solid-state distance relays. Figure 6 describes the manner in which advantage was taken of this set of inputs to identify the direction and location of a phase fault. V_{XY} was chosen to be the operating voltage and V_{ZY} the restraint voltage. In the circuit it can be seen that a "positive-going" V_{OP} leading a "positive-going" V_{RES} will cause the zener to breakdown and turn on the final transistor stage to provide a trip output. If V_{RES} leads V_{OP} in this circuit, the thyristor conducts first, preventing the voltage from reaching the zener breakdown level. This describes the essence of the concept, but the design artistry to assure proper operation required a great deal of finesse. To avoid as much as a cycle of delay for unfavorable fault incidence angle, this circuit was duplicated to accommodate the other half cycle. Also, once tripping was established, the thyristor gate had to be blocked to keep from losing the trip signal.

This simplified circuit of figure 6 does not do justice to the actual configuration, but it does show the basic concept of this type of successful distance relay.

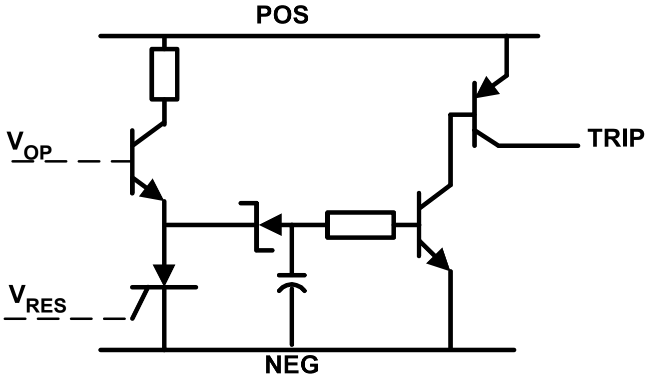
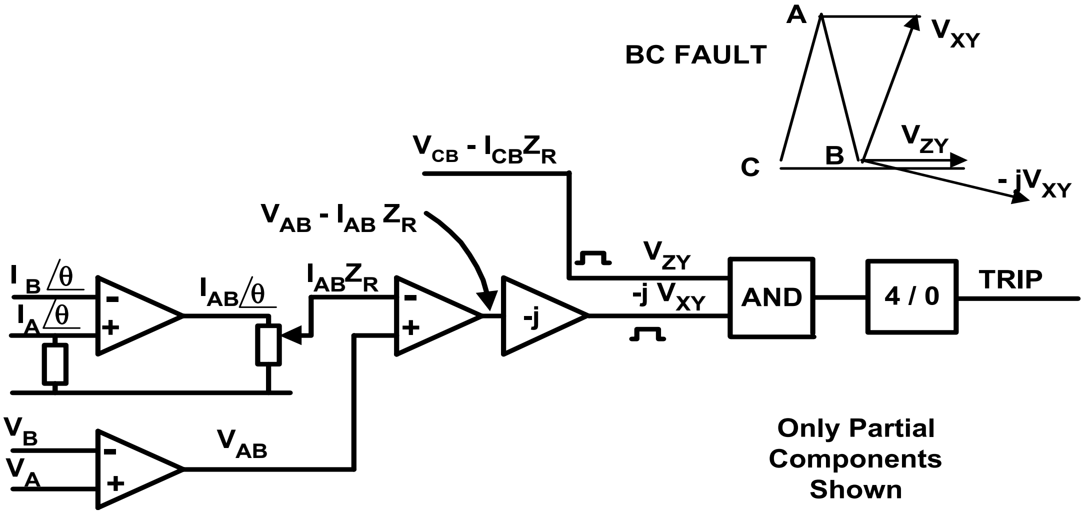


Figure 6 Solid-State Distance Relay

Operational Amplifier Distance Relay

Retaining a similar concept with the advent of integrated circuit relays, the distance relay took on the form of figure 7. In terms of power system quantities, the results are the same as that achieved with previous concepts. In this arrangement, the V_{XY} quantity is shifted by -90 degrees, and the key to operation is positive coincidence at the input to the "and" (or negative coincidence in the other half of the circuit, not shown) for a period of at least 4 milliseconds. The quantities jV_{XY} and V_{ZY} may have a phase separation of 90 degrees and still satisfy the 4 ms coincidence criterion. As in the previous cases, a forward phase-to-phase fault within the reach of the relay will produce a trip output. All reverse faults will be ignored. Some forward phase-to-ground, and some phase-to-phase-to-ground faults will be recognized. Load and swings will be ignored.



Only Partial
Components
Shown

Figure 7 Operational Amplifier Distance Relay

Microprocessor-based Distance Relay

The advent of the microprocessor led to a lower-cost method of implementing massive logical and arithmetic manipulation. Initially, the methods described above were translated to microprocessor use by creating phasor magnitudes and angles from the digital samples. Identification of the need to trip was accomplished **in exactly the same way as before**, by determining which phasor led and which phasor lagged.

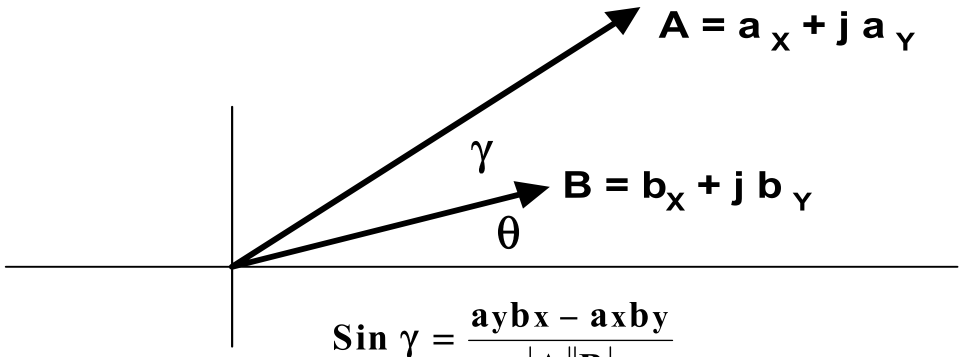
In most digital relays, the phasors are developed by taking advantage of simple trigonometric relationships. Multiplication of two in-phase sine waves having the same frequency and summing over a full period produces a positive result. If those two sine waves are displaced in time, the summation may be positive or negative (with the exception of a 90 degree displacement which sums to zero). Multiplication of two sine waves having different frequencies (one being an integral multiple of the other) and summing over a full period of the lower-frequency equals zero. A sine wave multiplied by a cosine wave and summed over a full period equals zero. Appendix I describes these relationships.

From these fundamental considerations, it can be seen that an arbitrary table of samples of a unit sine function can be multiplied by the similarly displaced selected samples of a voltage or current. If, then, these products are summed over a full period, a quantity proportional to the sine component of the sampled waveform can be found.

A similar process can be accomplished using a table of samples of a unit cosine function and multiplying them by the similarly displaced samples of the current or voltage. Summing these products over a full period gives a quantity proportional to the cosine component of the sampled waveform. From the sine and cosine components, the magnitude and angle of the voltages and currents can be determined, as described in Appendix I.

With the magnitude and angles of the voltages and currents known, the product of current and impedance setting can be determined and subtracted from the appropriate voltage to develop V_{XY} and V_{ZY} as was done in the previous cases. Appendix II shows that two phasors can be compared and one (called A) can be identified as **leading** the other (called B) if the y component of A multiplied by the x component of B is greater than the product of the x component of A and the y component of B. As before, if the operating phasor leads the polarizing phasor, the unit trips. If otherwise, the unit restrains.

One variety of microprocessor distance relay uses this comparison to identify whether or not tripping is the appropriate action. V_{XY} corresponds to phasor A and V_{ZY} corresponds to phasor B, The characteristic circle achieved with this implementation is identical to that described in figure 5.



$$\text{Sin}\ \gamma = \frac{aybx - axby}{|\mathbf{A}||\mathbf{B}|}$$

A leads B if sin γ is positive
A leads B if $a_Y b_X > a_X b_Y$

Figure 8 One Method to Determine Which Phasor Leads

Another Microprocessor Distance Relay Concept

It was recognized that increased speed of fault recognition could be realized if the initial impulse toward operation could be identified rather than actually waiting for the completion of the movement. This is equivalent to observing the magnitude and direction of torque on a cylinder unit without the speed, drift or friction drawbacks The actual design implements some representation of travel related to "energy level". Satisfying this concept with security against undesired operation required careful thought and laboratory verification.

As with the cylinder unit, maximum operating "energy" is present when the two phasors are at 90 degrees with respect to one another. At other angles, the net effect is related to the sine of the angle between them. V_{XY} and V_{ZY} are defined in the same way as is used throughout this paper.

In the cylinder unit, the fluxes are produced by the currents in the coils. The currents in the cylinder itself are produced by the derivative of these time varying fluxes. Torque is produced by the interaction between each flux and the currents that are produced by each adjacent pole flux.

It is observed in Appendix III that ϕ_{XY} produces I_d and I_f, and that ϕ_{ZY} produces I_e and I_g. Since the surface of the cylinder is uniform, then $I_d = I_f$ and $I_e = I_g$. Also, it is observed, for the 90 degree case shown, that I_e is in phase with ϕ_{XY} , but I_d is 180 degrees out of phase with ϕ_{ZY}.

Using the symbol ∝ for "proportional to", and using the products of Appendix III , this leads to**:**

$$\text{Torque} \propto Ie\,\phi_{XY} - I_d\,\phi_{ZY}$$

$$\text{but } Ie \propto d/dt\,(\phi_{ZY}), \text{ and } Id \propto d/dt\,(\phi_{XY})$$

$$\therefore\ \text{Torque} \propto \Big[\,(\phi_{XY})d/dt\,(\phi_{ZY})\Big] - \Big[(\,\phi_{ZY})d/dt(\phi_{XY})\Big]$$

This may be written:

$$\mathbf{Torque} \propto \Big[\,(\mathbf{V_{XY}})\mathbf{d/dt}\,(\mathbf{V_{ZY}}\Big] - \Big[(\mathbf{V_{ZY}})\mathbf{d/dt}\,(\mathbf{V_{XY}})\Big]$$

This states that torque on a cylinder unit can be described as the voltage V_{XY} times the derivative of V_{ZY} with respect to time minus V_{ZY} times the derivative of V_{XY}. An angle of 90 degrees has been assumed between the two quantities in appendix III. At angles other than 90 degrees, the magnitude of the torque is proportional to the sine of the angle by which V_{XY} leads V_{ZY}, and its direction is in the **operate (trip)** direction if the sine of the angle is positive. Translating this to microprocessor technology, the identical criterion can be applied.

Digitizing the quantities and the derivatives, and applying this concept, a sense of direction to a fault can be found in a very few samples. Thus, at **<u>very high speed</u>** the equivalent of **<u>magnitude and direction of equivalent torque</u>** can be identified and a trip or restraint condition established. Additional constraints are introduced to assure security of this digital concept. This concept has been applied for several years, providing reliable operational decisions within a half cycle. The fundamentals of establishing the magnitudes and derivatives based on the sampled quantities are described in Appendix IV.

Conclusions

While this paper only touches on some of the various concepts that have been used historically, it is hoped it will clarify the fundamental measurement properties of several that have been applied successfully. It is hoped that the reduction of these relaying principles to their bare-boned essentials will not mislead anyone to suggest that any of the components in the actual device be eliminated. They are there for a purpose. Also it is hoped that any inherent flaws in the simplified designs are not taken to mean that an oversight exists in the actual designs. All of the relays described have been in successful operation for a considerable time, and any deficiencies have been identified and corrected.

Acknowledgement

The counsel and advice of Elmo Price and Liancheng Wang of ABB Automation Inc., Allentown, Pa. in the preparation of this paper is acknowledged and was greatly appreciated.

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Appendix I

Use can be made of the following relationships :

$$\int_0^{2\Pi} \sin n\omega t \sin n w t \neq 0$$

$$\int_0^{2\Pi} \sin \omega t \sin n w t = 0 \qquad n \neq 1$$

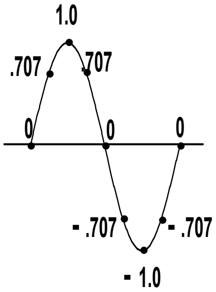
$$\int_0^{2\Pi} \sin \omega t \cos \omega t = 0$$

$$\int_0^{2\Pi} \sin n\omega t \cos n\omega t = 0$$

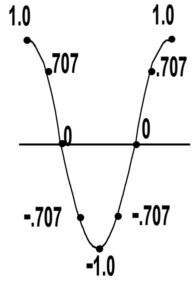
$$\int_0^{2\Pi} \cos \omega t \cos(\omega t + \phi) \neq 0 \qquad \phi \neq 90°$$

$$\int_0^{2\Pi} \sin \omega t \sin(\omega t + \phi) \neq 0 \qquad \phi \neq 90°$$

The last two expressions are the key to the use of the Fourier transform to convert a collection of samples into a phasor quantity. In the last ,using instead of sin ωt, a group of values displaced by an angle corresponding to the sampling rate of the processor (as for example, sine values displaced by 45 °, for a sampling rate of 8 samples per cycle), they may be multiplied by the samples of the pertinent sine-wave and the products summed over a full cycle. This gives a value proportional to the sine component of the waveform being sampled. Through a similar process with the next to last expression, the cosine component can be extracted from the samples using cosine values displaced by the same angle as before, With the sine and cosine components available, the magnitude and angle of the phasor corresponding to the samples can be determined.



sine coefficients



cosine coefficients

$$X r = \frac{2}{N} \sum_{i=0}^{N-1} x(i) \cos\left(\frac{2\Pi}{N} i\right) \qquad \text{REAL}$$

$$X i = \frac{2}{N} \sum_{i=0}^{N-1} x(i) \sin\left(\frac{2\Pi}{N} i\right) \qquad \text{IMAGINARY}$$

$$|V| = \sqrt{Xr^2 + Xi^2} \qquad \theta = \tan^{-1}\frac{Xi}{Xr}$$

Appendix II

Figure 8 describes two phasors with components $a_X + j\, a_Y$ and $b_X + j\, b_Y$. The purpose of this appendix is to show that a straightforward method exists to determine which phasor leads the other.

Forming the products $a_Y b_X$ and $a_X b_Y$ and subtracting the latter from the former, we get :

$$a_y b_X - a_X b_Y$$

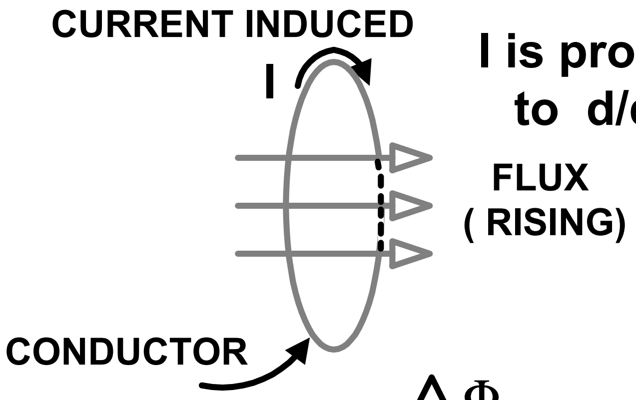
$$
\begin{aligned}
&|A| \sin(\gamma + \theta)\, |B| \cos\theta - |A| \cos(\gamma + \theta)\, |B| \sin\theta \\
&= |A||B| \left[\ \sin\gamma \cos\theta + \cos\gamma \sin\theta\ \right] \cos\theta \\
&\quad - |A||B| \left[\cos\gamma \cos\theta \sin\theta - \sin\gamma \sin\theta \sin\theta\ \right] \\
&= |A||\mathbf{B}| \sin\gamma \left[\ \cos^2 + \sin^2\theta\ \right] \\
&\quad + |A||\mathbf{B}| \cos\gamma \left[\ \sin\theta \cos\theta - \cos\theta \sin\theta\ \right] \\
&= |A||\mathbf{B}| \sin\gamma
\end{aligned}
$$

$|A|$ and $|B|$ are always positive and

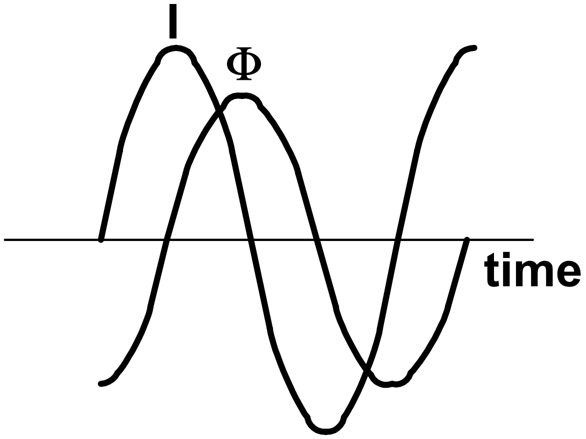
γ is positive if $\sin\gamma$ is positive and $< 180^\circ$

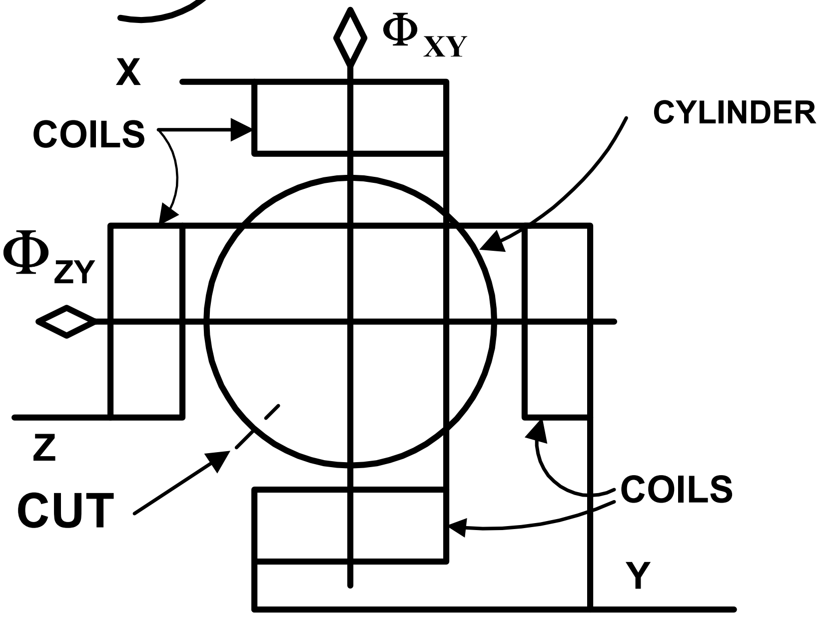
Conclusion : A leads B if $a_Y b_X - a_X b_y$ is positive

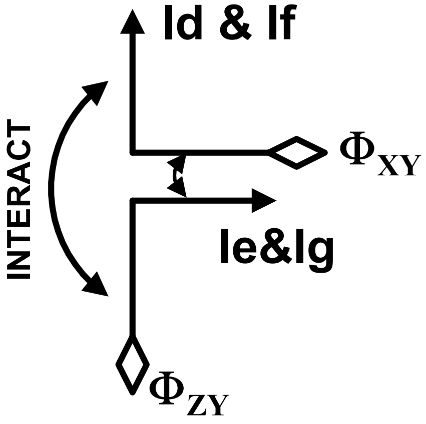
APPENDIX III DETAILS OF CYLINDER UNIT

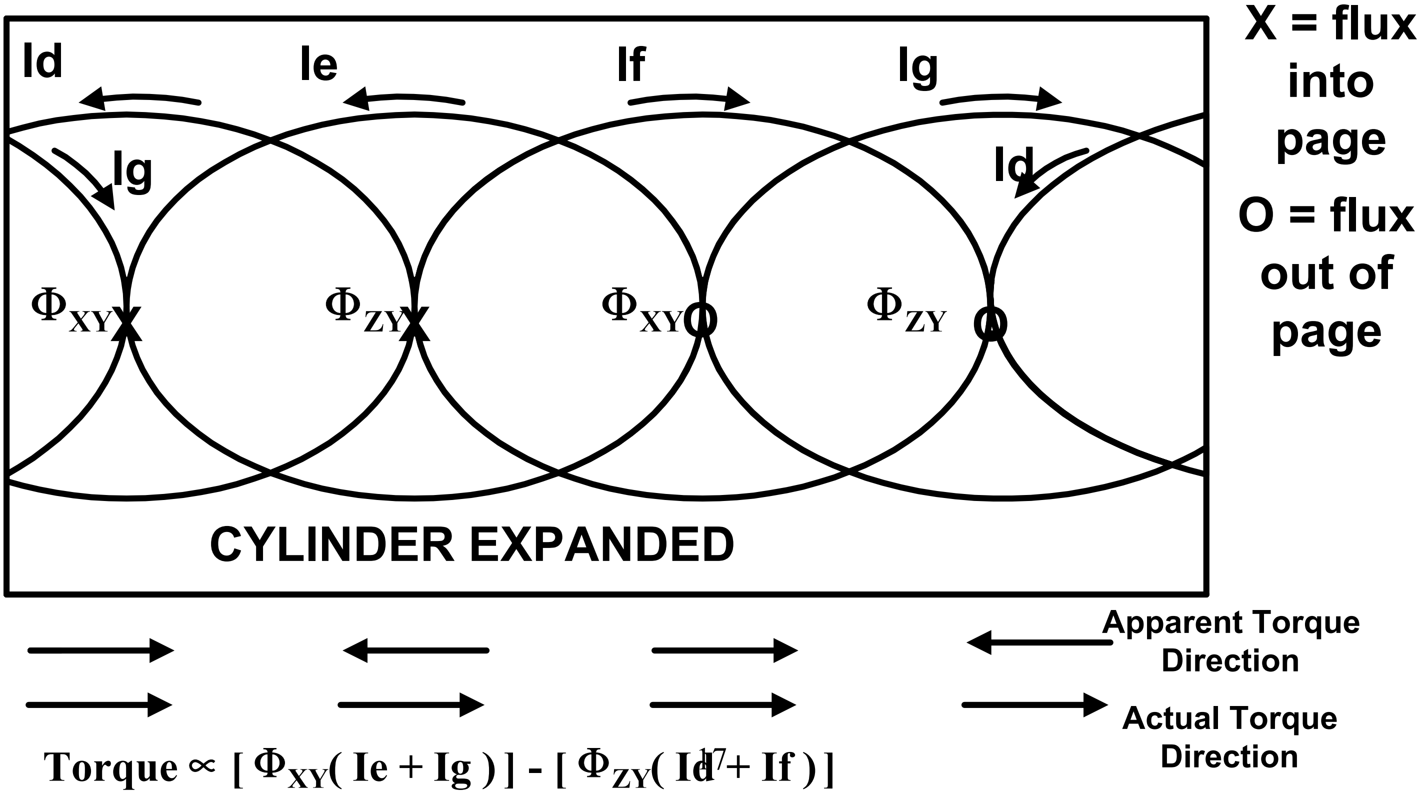


I is proportional to d/dt (flux)







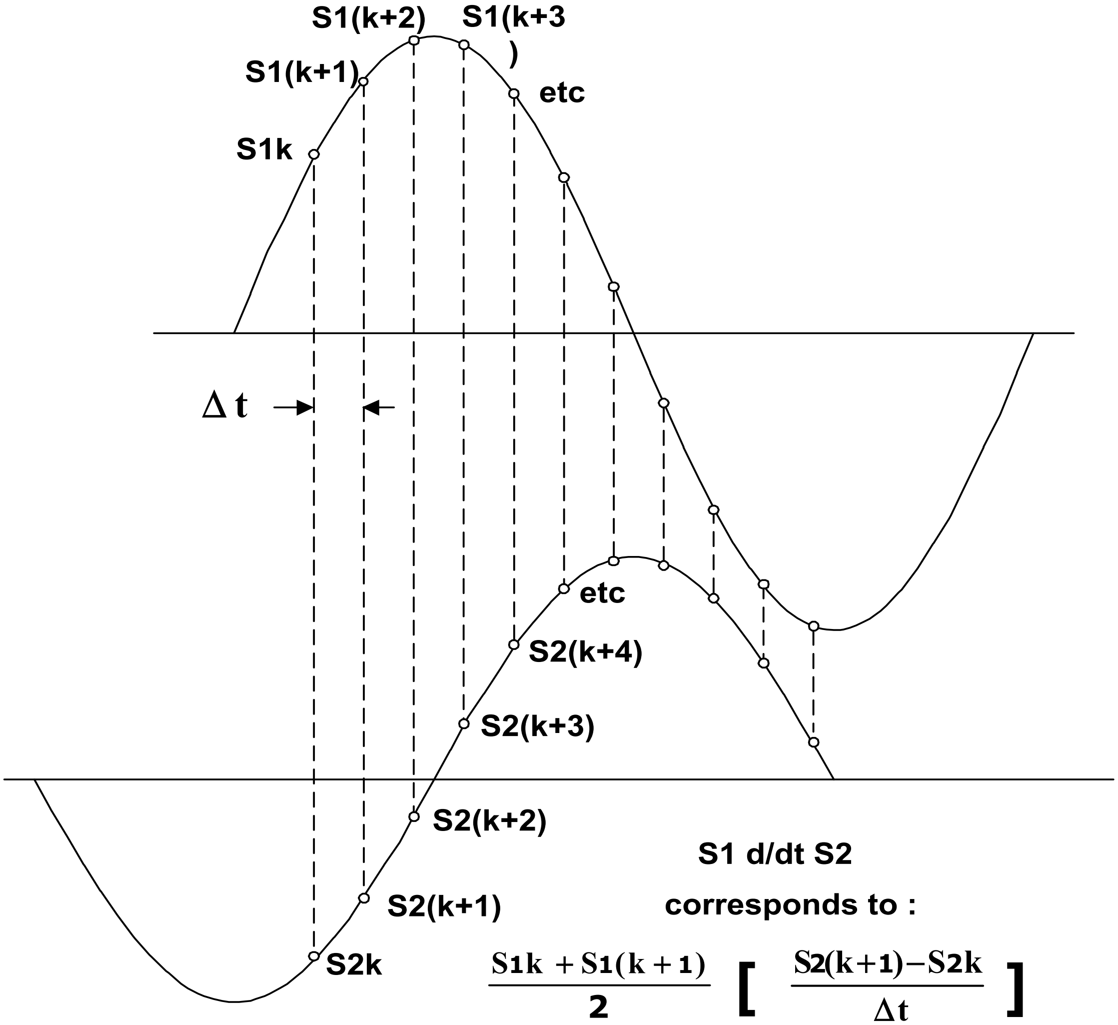


$$\text{Torque} \propto [\,\Phi_{XY}(\,Ie + Ig\,)\,] - [\,\Phi_{ZY}(\,Id + If\,)\,]$$

$$\text{Torque} \propto V_{XY}\,d/dt\,V_{ZY} - V_{ZY}\,d/dt\,V_{XY}$$

Appendix IV

Considering the expression**: Torque ∝ [Vxy d/dt Vzy] - [Vzy d/dt Vxy]** and viewing the two sine-waves with an assumed 90° relationship, the method by which the individual samples may be used to determine the result of this equation may be seen:



S1 d/dt S2

corresponds to :

$$\frac{S1k + S1(k+1)}{2} \left[\frac{S2(k+1) - S2k}{\Delta t} \right]$$

Letting S1 = Vxy and S2= Vzy, it can be seen how Vxy d/dt Vzy - Vzy d/dt Vzy can be obtained and a trip decision made

Biographical Sketch

Walter A. Elmore was born in Bartlett, Tennessee, served in the Army Air Corps as a navigator during World War II, and graduated from the University of Tennessee with a B.S.E.E. in 1949. He was in Substation Design at Memphis Light Gas & Water Division until he joined Westinghouse in 1951 as a District Engineer in Seattle, Washington. He transferred to the Relay-Instrument Division in Newark, New Jersey in 1964, where he became Manager of the Consulting Engineering Section. He held that position, following a 1989 merger with ABB, until 1992 in Coral Springs, Florida. From 1992 until 1996, when he retired, he held the position of Consulting Engineer. He continues to work as a consulting engineer for ABB. In August 1996, he had the great honor of having the ABB manufacturing plant in Coral Springs, Florida dedicated to him.

He is past chairman of the IEEE / PES Technical Council, and past chairman of the IEEE / PES Power System Relaying Committee. He is a Life Fellow of the IEEE, and was presented the IEEE Gold Medal for Engineering Excellence in 1989. He was accepted as a member of The National Academy of Engineering in 1998.

He has presented over 100 technical papers, is one of the authors of the Year 2000" Standard Handbook for Electrical Engineers," and is the editor and co-author of two books: "Protective Relaying Theory and Applications" and " Pilot Protective Relaying."

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