# Zero Sequence Mutual Effects on Ground Distance Relays and Fault Locators

by

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## Introduction

Zero sequence mutual effects have been a source of concern for many years and extensive studies have been conducted. This paper will attempt to describe this phenomenon in basic terms and to provide some assistance in evaluating the severity of its influence on ground distance relays and fault locators.

The fundamental criteria for distance relays are different from those for fault locators. Distance relays require an immediate identification of a fault only as being within or outside of their protective zone. How close the fault may be to the measuring terminal or to the boundaries of the distance relay's protective zone are of no pertinence to the basic function of a distance relay, which is to trip or block.

The fault locating function, which may be an inherent part of a modern microprocessor relay or be an independent device, is not encumbered by the requirement for a quick decision, but it must identify with a high degree of certitude where a fault is located and as clearly as possible, its type.

Many factors other than mutual impedance have an unfavorable influence on distance measurement and fault location, but only the effect of mutually induced voltage is discussed here.

## The Fundamentals

Figure 1 describes the process of mutual induction in its simplest form. Current in one conductor induces a voltage in the adjacent conductor that is proportional to current magnitude, length of proximity and mutual impedance per unit length. The mutual impedance is related to $D_e$, the depth of earth return and the GMD, geometric mean distance between the conductors. $D_e$ is defined as $2160\sqrt{\rho/f}$ where $\rho$ is earth resistivity in ohms per meter cubed, and f is frequency in hertz. This is based on Carson's formulas [1].

In Figure 1 the voltage $V_S$, required to drive 1 ampere of current through conductor A is the self impedance of the circuit in ohms and voltage $V_m$ induced in circuit B for this condition is the mutual impedance. Zero sequence mutual impedances are based on earth return, and therefore, the earth resistance must be included as an inherent part of the impedance.

When considering three-phase circuits, the impedance of Figure 1 applies to the mutual impedances between phases. From a positive and negative sequence viewpoint, with the phase currents being 120° apart, it can be shown from the configuration of Figure 2, for example, that, for a transposed circuit with a full roll, each conductor in each circuit alternately occupies the same space as the other conductors in its group, making all distances between adjacent groups, on average, the same. This causes the positive and negative sequence mutual impedance between circuits to be zero. On the other hand, there will be a positive and negative sequence mutual between circuits if the circuits are not transposed within the group and with respect to each other.

60 Hertz Mutual Values per Mile

$$\begin{cases} r_m = 0.0954 \\ x_m = 0.2794 \log \dfrac{D_e}{GMD} \\ D_e = 2160\sqrt{\rho/f}\,, \quad \rho = \text{Meter Ohms Earth Resistivity} \\ \qquad\qquad\qquad\quad\;\; f = \text{Frequency} \end{cases}$$

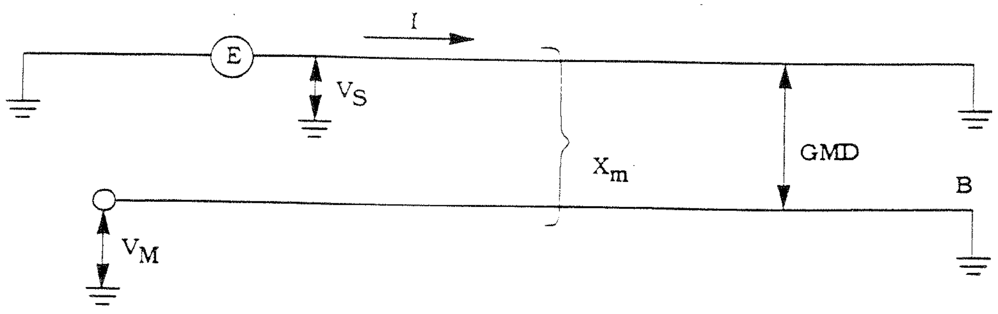
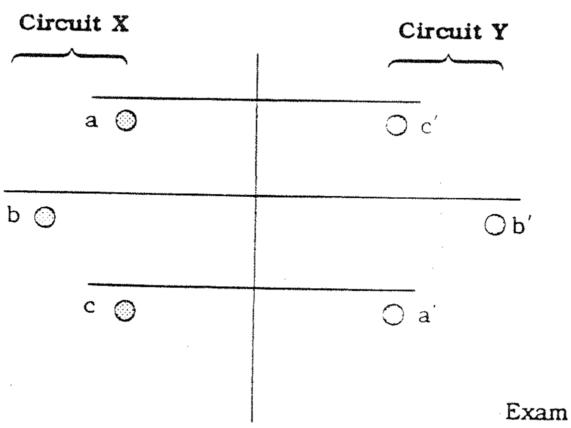


Figure 1. Basic Concept of Mutual Impedance.

Zero sequence mutual on the other hand is quite different from positive and negative sequence mutuals because of the fact that currents $I_{ao}$, $I_{bo}$, and $I_{co}$, are equal in magnitude and in phase with one another. Transpositions do nothing to minimize zero sequence mutual. However, their lack causes sequence interaction to take place and zero sequence voltage to result from the flow of positive or negative sequence current.

Zero sequence mutual between 3-phase circuits or three phase circuits and other independent circuits such as one or more overhead ground wire can be determined by analytical methods as described in Wagner and Evans "Symmetrical Components" [2]. The zero sequence configuration is generally shown as in Figure 3. Each circuit may be treated as a lumped element representing any number of conductors. With impedance in the zero-sequence network to be treated on a "per phase" basis, the equations of Figure 1 must be multiplied by 3, giving $Z_m = 0.286 + j\,0.8382 \log De/GMD$ ohms per phase per mile. $GMR_X$ and $GMR_Y$ are used in determining the "self" impedance of the two circuits.



Voltage Drops Per Unit Distance

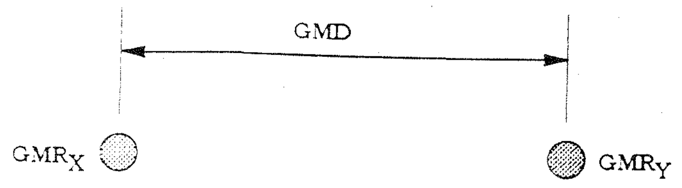
$$V_A = I_a Z_{aa} + I_b Z_{ab} + I_c Z_{ac} + I_a' Z_{aa}' + I_b' Z_{ab}' + I_c' Z_{ac}'$$

$$V_b = I_a Z_{ba} + I_b Z_{bb} + I_c Z_{bc} + I_a' Z_{ba}' + I_b' Z_{bb}' + I_c' Z_{bc}'$$

$$V_c = I_a Z_{ca} + I_b Z_{cb} + I_c Z_{cc} + I_a' Z_{ca}' + I_b' Z_{cb}' + I_c' Z_{cc}'$$

Examples: $Z_{aa}$ = self impedance
$Z_{ab}$ = mutual impedance between a and b

Figure 2. Typical Twin Circuit Tower.



GMR$_X$ = Geometric Mean Radius of all 3 conductors of circuit X

GMR$_Y$ = Geometric Mean Radius of all 3 conductors of circuit Y

GMD = Geometric Mean Distance between circuits X and Y

Figure 3. Equivalent Zero Sequence Network for the Circuits of Figure 2.

## Ground Distance Relaying

Many relaying concepts are used in identifying whether a fault is within the protected zone of a relay or not. All have their strengths and weaknesses, none being perfect in all categories, some excelling in several.

This paper focuses on one widely used ground relaying principle that eliminates several of the major errors. It uses:

$$V_{\phi G} - (I_\phi + KI_0)\,Z_C \qquad (1)$$

as an operating quantity and $V_Q$ as a polarizing quantity. For "A-phase," $V_{\phi G}$ is $V_{AG}$ at the relay location, $I_\phi$ is $I_A$, K is a constant $(Z_{0L}/Z_{1L}-1)$ with $Z_{0L}$ and $Z_{1L}$ being zero and positive sequence impedance of the transmission line, $I_0$ being zero sequence current in the protected line $Z_C$ the setting of the relay in terms of the desired positive sequence reach, and $V_Q$ being $V_{CB}$ in this case.

Many references, (including "Applied Protective Relaying," [3] contain a derivation for voltage at a point remote from an A-phase to ground fault (See Figure 4):

$$V_{AG} = mZ_{1L}\left[I_A + \frac{Z_{0L} - Z_{1L}}{Z_{1L}}K_0 I_0\right] + 3I_0R_F + I_{0E}\,m\,Z_{0M} \qquad (2)$$

where:

$V_{AG}$ = A to ground voltage at station I

m = Per unit distance to the fault (1 per unit = fault at station II)

$Z_{1L}$ = Positive sequence impedance of the line between breakers A and B.

$Z_{0L}$ = Zero sequence impedance of the line between breakers A and B.

$I_A$ = Phase current in breaker A for an AG fault at m per unit.

$Z_{1E}$ = Positive sequence impedance of the line between breakers C and D.

$Z_{0E}$ = Zero sequence impedance of the line between breakers C and D.

$K_0$ = Distribution factor describing the per unit value and angle, relating the zero sequence current in breaker A to the total $I_0$ in the fault. $K_1$ is the same except for positive sequences.

$P_0$ = Same as $K_0$ except it is per unit zero sequence current in the source. $P_1$ is the same except it is per unit positive sequence current in the source.

$R_F$ = Effective fault resistance.

$I_{0E}$ = Zero sequence current in the parallel line.

$Z_{0M}$ = Total zero sequence mutual impedance (assumed uniform throughout the length of the line) between line AB and line CD.

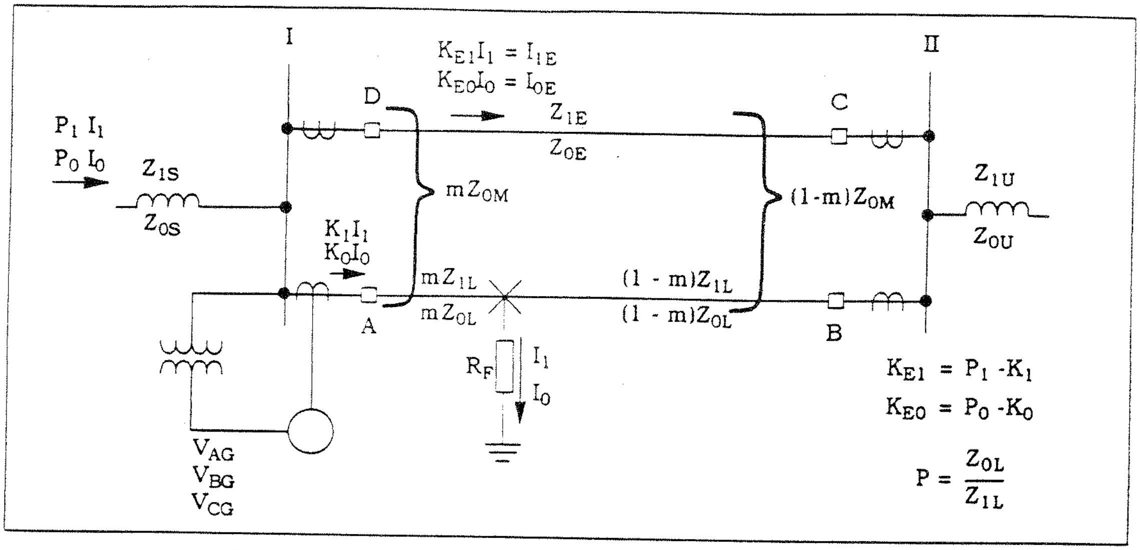


Figure 4. Typical One Line Diagram with Parallel Line.

Equation 2 is completely general for the $\phi_G$ (phase-to-ground) fault, including out of phase sources, fault resistance, zero-sequence mutual and any combination of line and source impedances.

One popular method of determining m, the per unit distance to a phase A-to-ground fault is through a determination of:

$$Z_{APP} = \frac{V_{AG}}{I_R} \qquad (3)$$

where $Z_{APP}$ = apparent impedance, $I_R = I_A + \left[\frac{Z_{0L} - Z_{1L}}{Z_{1L}}\right] K_0 I_{A0}$ and $V_{AG}$, is as previously defined.

Using equation (2) and substituting $K = \frac{Z_{0L} - Z_{1L}}{Z_{1L}}$ (note K has magnitude and angle), it is seen that:

$$\frac{V_{AG}}{I_R} = \frac{V_{AG}}{I_a + KK_0I_0} = mZ_{1L} + \frac{3I_0R_F}{I_A + KK_0I_0} + \frac{I_{0E}\,mZ_{0M}}{I_A + KK_0I_0} \qquad (4)$$

with the first quantity representing the actual distance to the fault, the second describing the error associated with fault resistance and the third being the error imposed by mutual.

It will be recognized that all of the quantities in the denominator of the second two terms are fixed or accessible to the relay, while the elements of the numerators are inaccessible and/or vary with fault location.

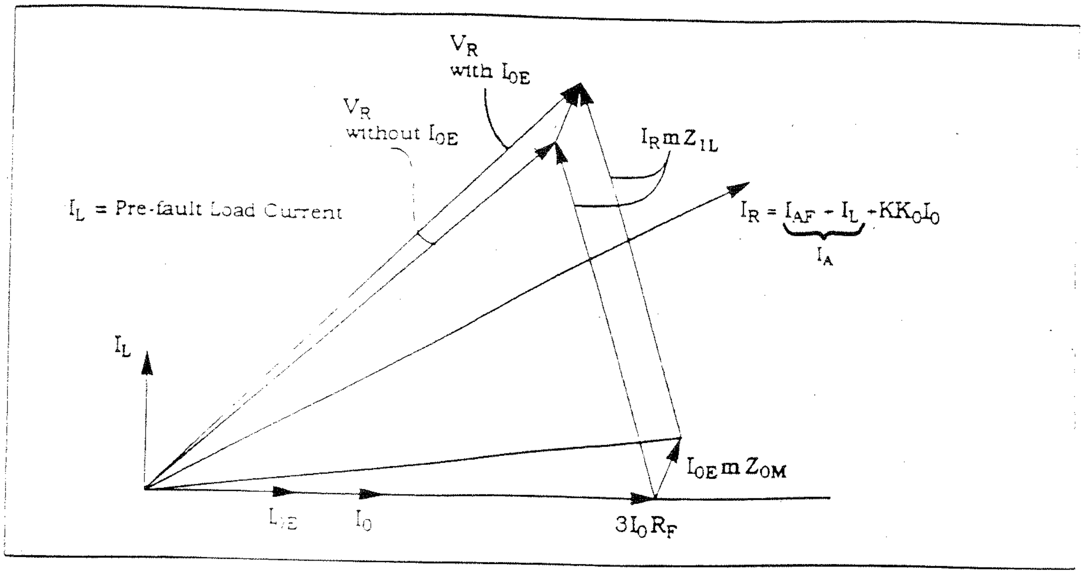


Figure 5. Phasor Diagram With and Without Mutual, With $R_F$.

The phasor diagram of Figure 5 is instructive and shows how these second two terms of equation (4) influence the relay voltage. The zero sequence current in the fault was chosen as a reference quantity for this diagram. $3I_0R_F$ is fault voltage, and with $R_F$ being purely resistive, appears also at a zero angle. $I_{0E}$ may be at approximately the same angle as $3I_0$ (but not necessarily) producing $I_{0E}mZ_{0M}$ at a high leading angle. The fault voltage and the mutually induced voltage are added together vectorially. To this sum is added $I_RmZ_{1L}$ giving $V_R = V_{AG}$ the voltage at the relay. With $I_{0E}$, in the direction shown, $V_R$ is higher than it would have been without mutual, causing distance relay underreach. With $I_{0E}$ in the opposite direction, as it would be for a nearby fault, overreach results. Figure 6 shows the phasor diagram without mutual or fault resistance.

Figure 7 was constructed from equation (4) after substituting $I_A + KK_0\,I_0 = I_0\,(K_1 + K_2 + pK_0) = I_0\,(2K_1 + pK_0)$ and $I_0(P_0 - K_0) = I_{0E}$. Note $p = Z_{0L}/Z_{1L}$. Figure 7 applies to any relaying or fault locating function, it being developed strictly from power system relationships without regard to any application.

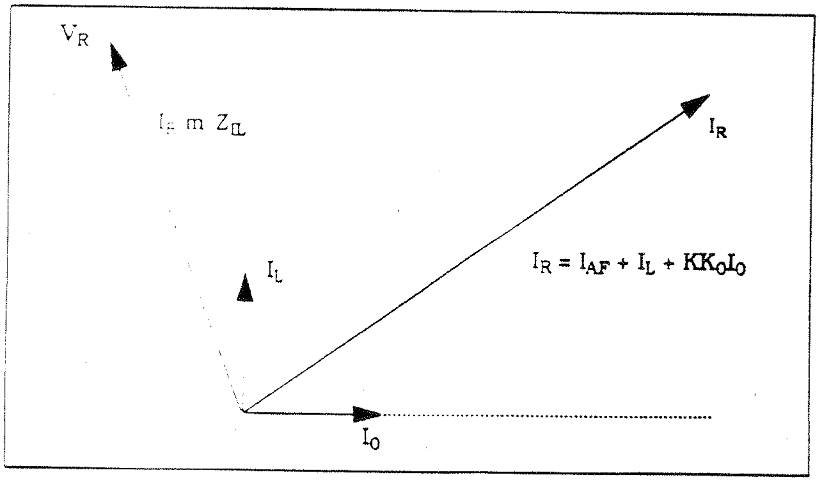


Figure 6. Phasor Diagram, No Mutual, No Fault Resistance.

Several things, some of which have been described profusely elsewhere, can be observed in Figure 7:

1. The fault resistance effect on the apparent impedance distinctly has a reactance component.
2. The apparent reactance of fault resistance may be positive or negative depending on pre-fault load current direction. The angle of tilt, $\alpha$, is dependent on the angle of $I_0/I_R$.
3. The zero sequence mutual effect can be in either direction and produces both a resistance and reactance change in the apparent impedance.
4. The mutual effect may nullify part of, or aggravate the influence of fault resistance.
5. $P_0$, $K_0$, $K_1$ are all influenced by fault location, m.

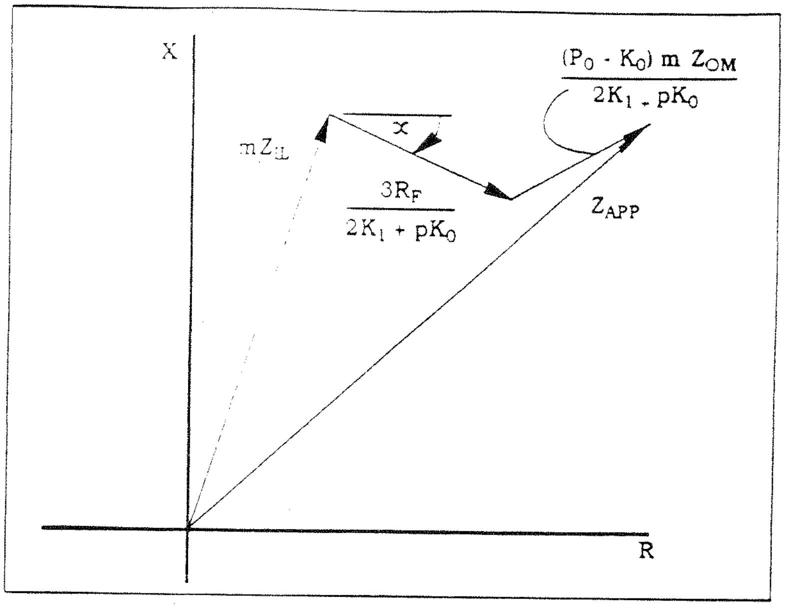


Figure 7. Apparent Impedance for A-G fault with $R_F$ and Mutual.

## Effect on Distance Relays

As Figure 7 shows, $Z_{APP}$, apparent impedance may be quite different from $mZ_{1L}$, the actual impedance to a fault. This will have different effects on distance relays depending on their principle of measurement.

Figure 8 points out an interesting criterion. For a fault at some location, m, per unit of the line length from the relay, no zero sequence current flows in the adjacent line (or collection of lines in parallel). Obviously, no zero sequence voltage difference exists between busses I and II for this case. The value of m at which this occurs is independent of the line and is dependent solely on $Z_{0S}/(Z_{0S} + Z_{0U})$, the source impedance ratio, where $Z_{0S}$ is zero sequence impedance behind bus I, and $Z_{0U}$ is zero sequence source impedance beyond bus II.

For a fault at $m = Z_{0S}/(Z_{0S} + Z_{0U})$, no error due to mutual exists in the impedance measurement. For a fault short of this point, the relay will have a tendency to identify the fault as being closer than it actually is. For faults beyond this point, the relay will see it to be farther away than it actually is as a result of the zero sequence mutual effect.

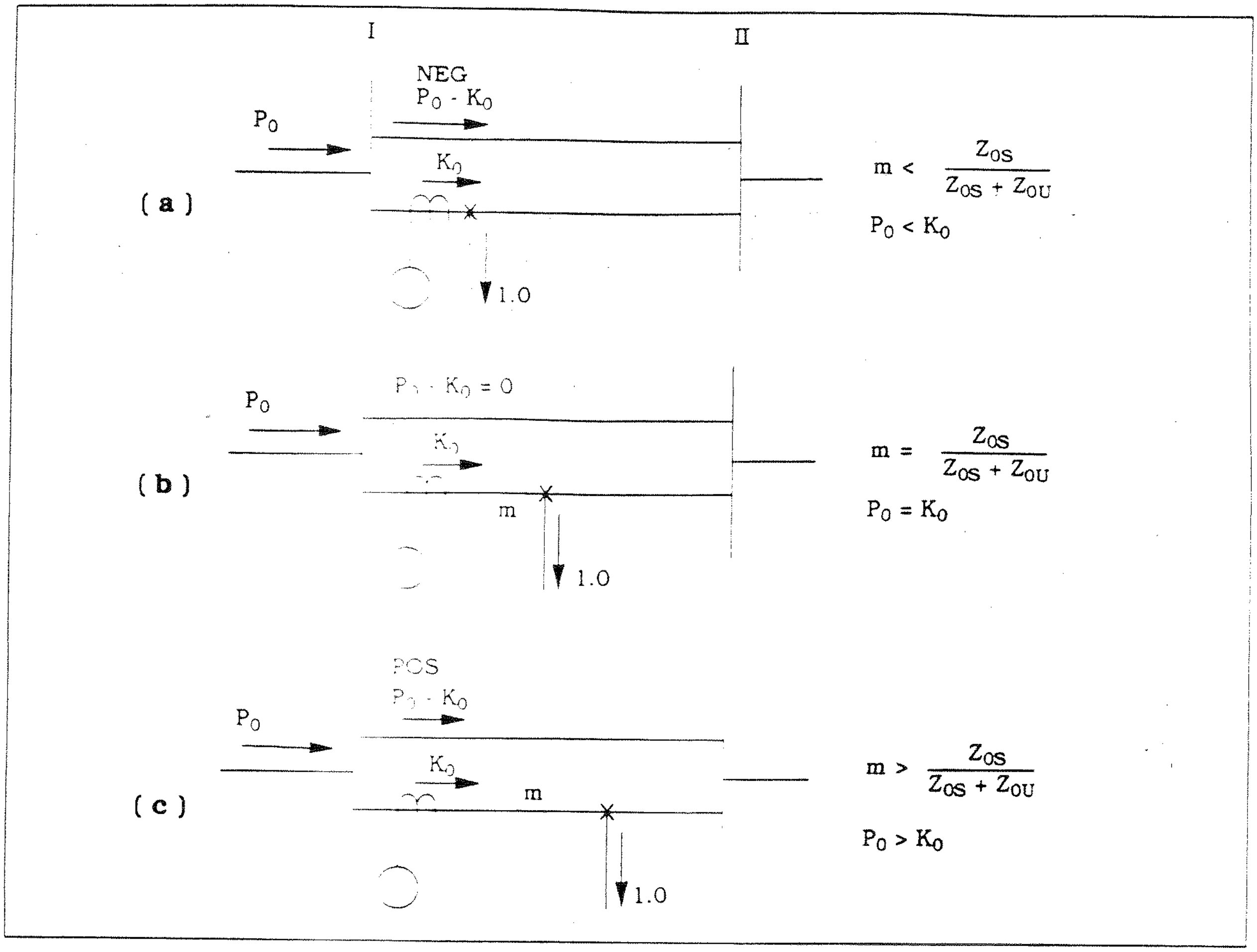


Figure 8. Effect on Adjacent Line Zero Sequence Current of Fault Location.

It can be seen in Figure 7 that the apparent impedance seen by a relay for a line-to-ground fault at m without $R_F$ fault resistance, is:

$$Z_{APP} = mZ_{1L} + \frac{(P_0 - K_0)\, m Z_{0M}}{2K_1 + pK_0} \qquad (5)$$

From this we can obtain:

$$\frac{Z_{APP}}{mZ_{1L}} = 1 + \frac{(P_0 - K_0)\left(\frac{Z_{0M}}{Z_{1L}}\right)}{2K_1 + pK_0} \qquad (6)$$

If it is now assumed that the parallel line and the protected line are identical and the fault is at m = 1, $P_0 - K_0$ will be equal to $K_0$ and equation (7) results.

$$\frac{Z_{APP}}{Z_{1L}} = 1 + \frac{K_0 \frac{Z_{0M}}{Z_{1L}}}{2K_1 + pK_0} \qquad (7)$$

Further, dividing numerator and denominator of the second term by $K_0$ yields equation (8).

$$\frac{Z_{APP}}{Z_{1L}} = 1 + \frac{Z_{0m}/Z_{1L}}{2K_1/K_0 + p} \qquad (8)$$

This is plotted in Figure 9. It is useful for determining the setting required for an overreaching ground distance relay based on known impedance values and distribution factors. The values from the curves are minimum settings that will barely allow the relay to reach the next bus with no allowance for anything but zero sequence mutual effects. Such settings would be used for overreaching pilot applications and for zone-2 time applications with due consideration given to other error sources including infeed effect and fault resistance. Acknowledgment is made to the work of G. D. Rockefeller in his paper [4] for the development of Figure 9.

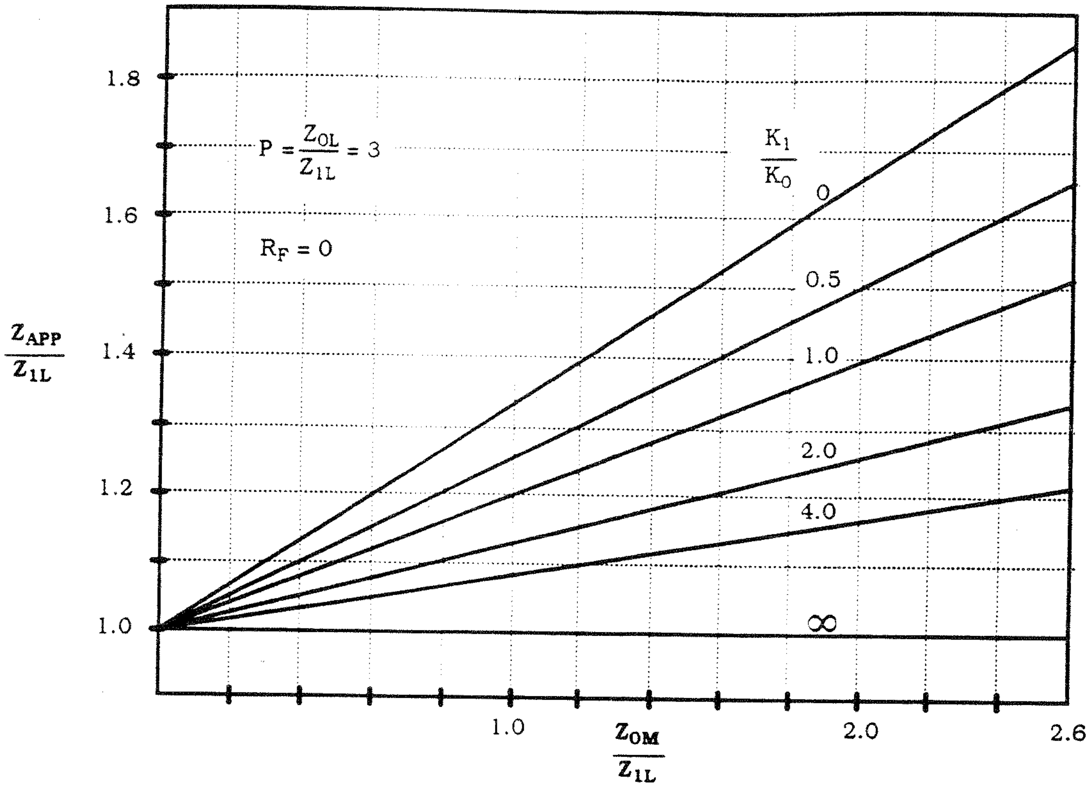


Figure 9. Apparent Impedance Seen By Ground Distance Relay for a øG Fault at1.0 per unit.

Going back to Figure 8a, where it was seen that the distribution factor for adjacent line current is $P_0 - K_0$, it is obvious that this current becomes negative when $m < Z_{0S}/(Z_{0S} + Z_{0U})$. When $P_0 - K_0$ becomes negative, (that is $P_0/K_0 < 1$) the impedance seen by the relay shown in Figure 8 is smaller than $mZ_{1L}$, the actual impedance to the fault. Figure 10 attempts to shed light on the effect on a zone-1 relay of various mutual impedances and fault current distribution factors.

Since the m at which this reversal occurs is $Z_{0S}/(Z_{0S} + Z_{0U})$, a small $Z_{0S}/Z_{0U}$ ratio causes the point of reversal to be very near the relay location. Most faults on the line, then, will cause an underreach tendency on the part of any ground distance relay used. For the extreme case of no zero sequence path to ground (source) at terminal I, $P_0/K_0$ will be zero for all faults on the protected line, and the relay shown will see a lower impedance than the actual impedance to a fault and will thus overreach the point to which it is set to reach.

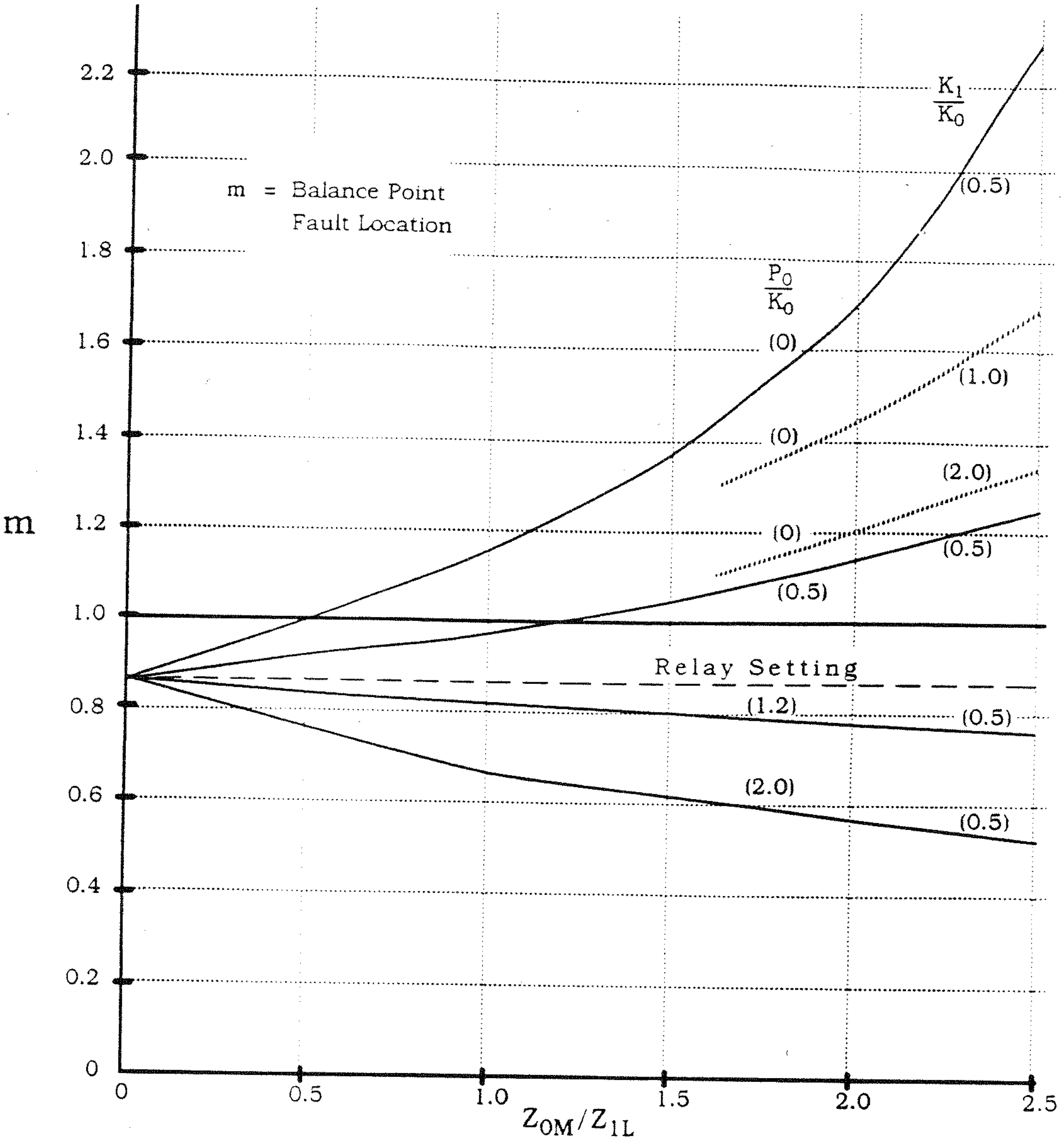


Figure 10. Influence of Zero Sequence Mutual on Reach of Zone-1 Relay Set for 0.85 $Z_{1L}$.

## Parallel Line Grounded for Maintenance

If a parallel line (having significant zero sequence mutual) is isolated and grounded at both ends for maintenance, as shown in Figure 11a, and a phase-to-ground fault occurs on the energized circuit, the zero sequence current in the protected line induces a voltage in the line which is isolated and grounded. This causes a zero sequence current to flow in the grounded line, which, in turn, induces a voltage in the protected line. The direction of the induced voltage is such as to cause the relay to reach farther than it would otherwise. Figure 11b attempts to describe this effect for the worst case of a fault at 1.0 per unit ($Z_{1L}$ positive sequence ohms away from the relay location).

For a long line application, one would expect a $K_1/K_0$ ratio to be greater than 1.0, though source impedances influence this greatly. With a severe zero sequence mutual, and a zone-1 ground relay setting of 85%, it can be seen from Figure 11 that the relay may operate for a ground fault on the next bus or slightly beyond it, under the condition of a grounded adjacent line.

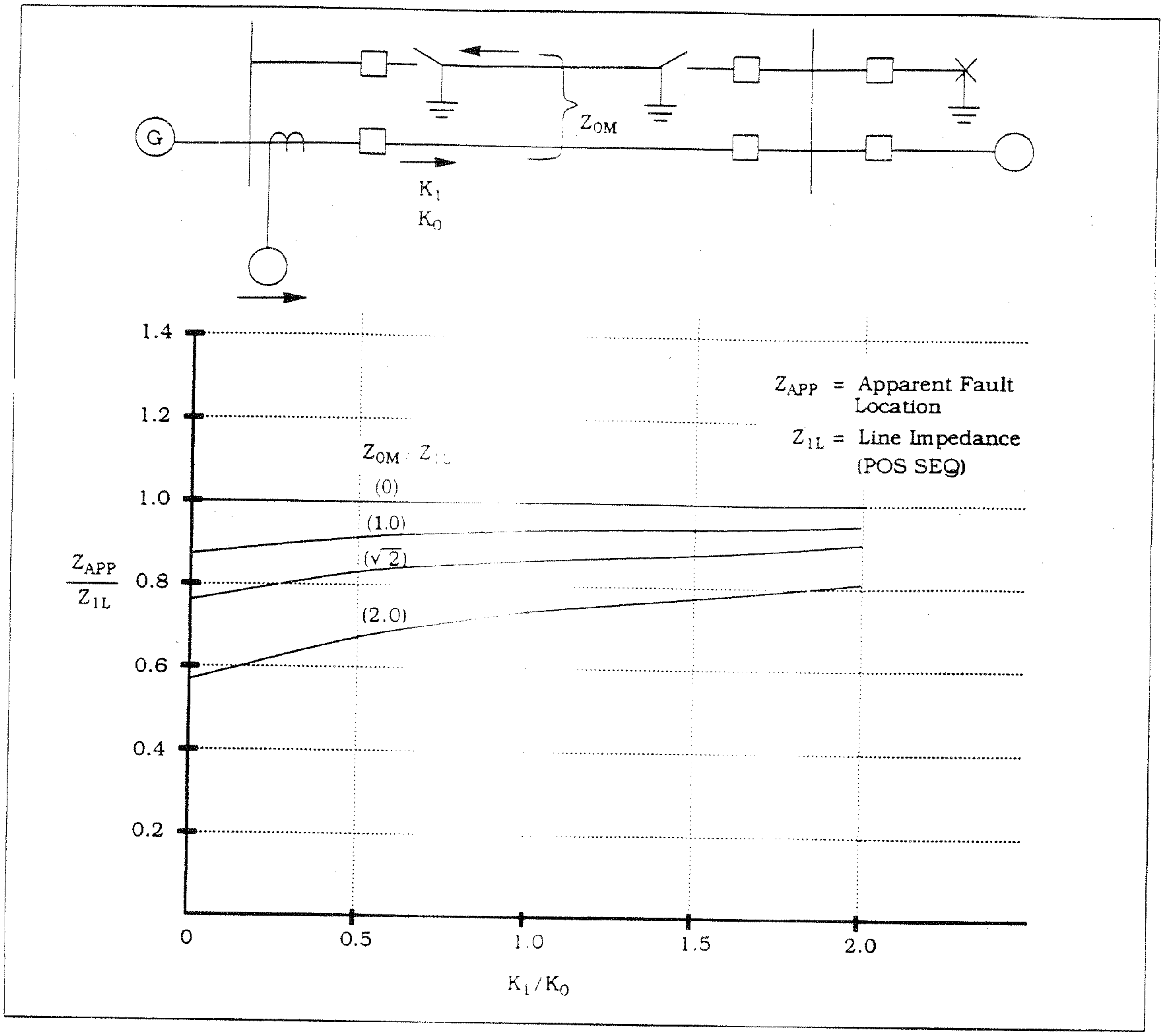


Figure 11. Effect on Apparent Distance to Fault at 1.0 Per Unit with Adjacent Line Grounded

## Phase-to-Phase-to-Ground Faults

The per unit error introduced by zero sequence mutual in the B phase distance measurement is, for a BCG fault:

$$\text{error} = (P_0/K_0 - 1)(Z_{0m}/Z_{1L})[(K_1/K_0)(a - q)/(1 - q) - p] \qquad (9)$$

where $a = -0.5 + j0.866$

$q = Z_{0T}/(Z_{0T} + Z_{2T})$ where $Z_{0T}$ and $Z_{2T}$ are total zero and negative sequence impedance

and $p = Z_{0L}/Z_{1L}$

This is substantially less than the error associated with the phase-to-ground fault. From this one case studied, it appears that phase-to phase-to-ground faults do not rule in terms of mutual influence on ground distance relays.

## Adjacent Line Faults

Figure 12 describes one of the important influences of zero sequence mutual. For the case of two identical parallel lines, an open breaker at D, a ground fault at D, and an open source at station II, a zone-2 relay at A may reach the full distance around the loop as a result of mutual and "see" this fault. This could be troublesome except for the fact that, under the influence of the same mutual, the zone-1 relay at C extends its reach to cover the entire line between C and the open breaker at D. Using a zone-1 setting of 0.85 $Z_{1L}$ at all locations, coordination between the two relays is maintained, irrespective of a large magnitude of the mutual impedance. With lower mutual impedances coordination is maintained. With a source at station II, coordination is also maintained.

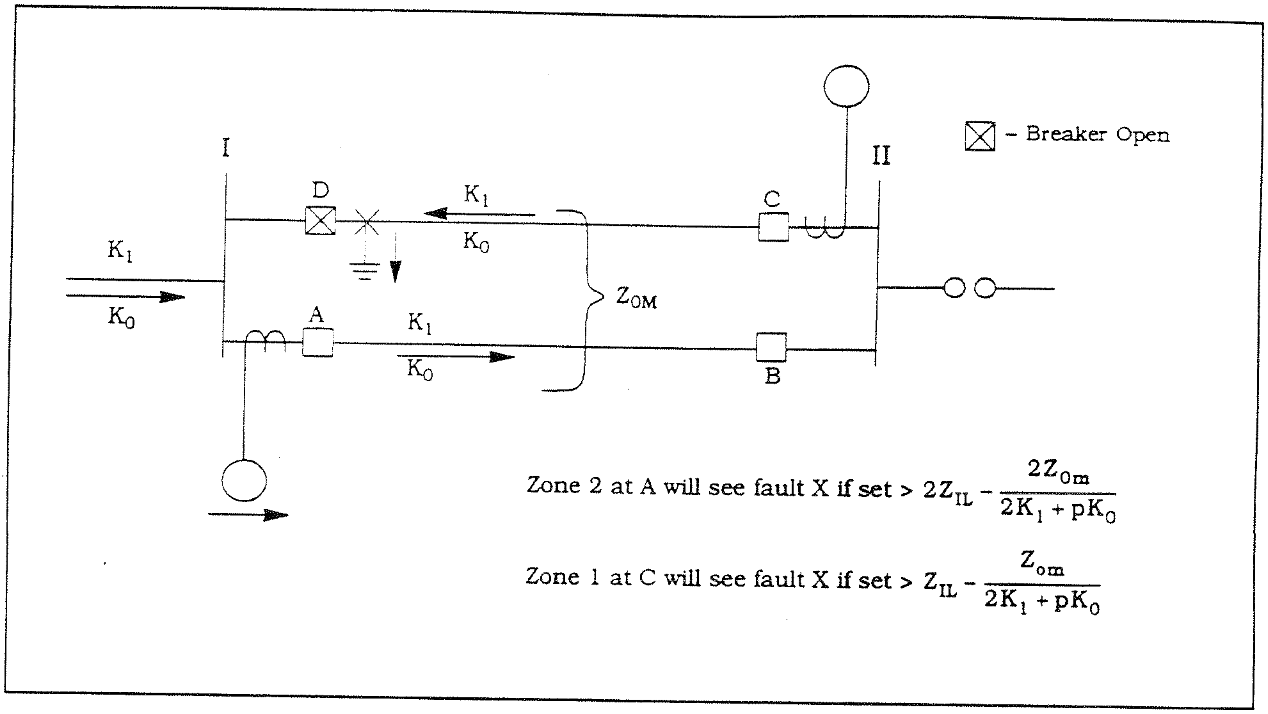


Figure 12.Effect on Zone 2 and Adjacent Line Zone 1 With Adjacent Line Ground Fault.

## Mutual Compensation

Where two transmission lines terminate at both ends, in common substations, it is possible to gain access to the adjacent line zero sequence current for introduction into the relay that is affected by its presence. However, it is evident from the discussion here that the significance of this current in a relaying measurement varies widely depending on fault location. To use it effectively, there must be an appraisal of fault location, then an evaluation of the significance of mutual for a fault at that location, then a reevaluation of the fault location. For relaying, this produces an intolerable delay for questionable benefit. For a fault locator, it may have some value in a simple, clean application as described above.

This is complicated by the fact that severe mutual influence may be present with the adjacent line breaker open. No access exists, then, to the adjacent line zero sequence current. Also, as described, grounding of the adjacent line can cause an appreciable influence, and again there is no access to the adjacent line current.

To accommodate the influence of multiple lines having zero sequence mutuals with the protected line may require a very complicated interconnection of current transformer circuits and a complicated evaluation algorithm, with debatable value.

For adjacent line faults, the mutual compensation may overpower the observed transmission line current and produce a false directional sense. To prevent this, logic must be incorporated to determine the relative magnitude of the adjacent line zero sequence current and that of the protected line to prevent overtripping.

The complication and cost of mutual compensation is generally not justifiable.

## Fault Locators

To identify the location of a fault subjected to all of the peculiarities described, is to identify m. To do it, with information available at only a single transmission line terminal is virtually impossible, but some methods provide a reasonably accurate estimate:

1. Law of sines: By utilizing various trigonometric relationships, the effect of zero sequence mutually induced voltage can be determined. From Figure 13, known values can be applied to determine $\alpha$ and $\beta$. Then $V_{AG}/\sin\beta = I_R m Z_{1L}/\sin\alpha$. The values of $\alpha$, $\beta$, $V_{AG}$, $I_R$ and $Z_{1L}$ are known allowing m, the per unit distance to the fault to be determined. This method may be applied to

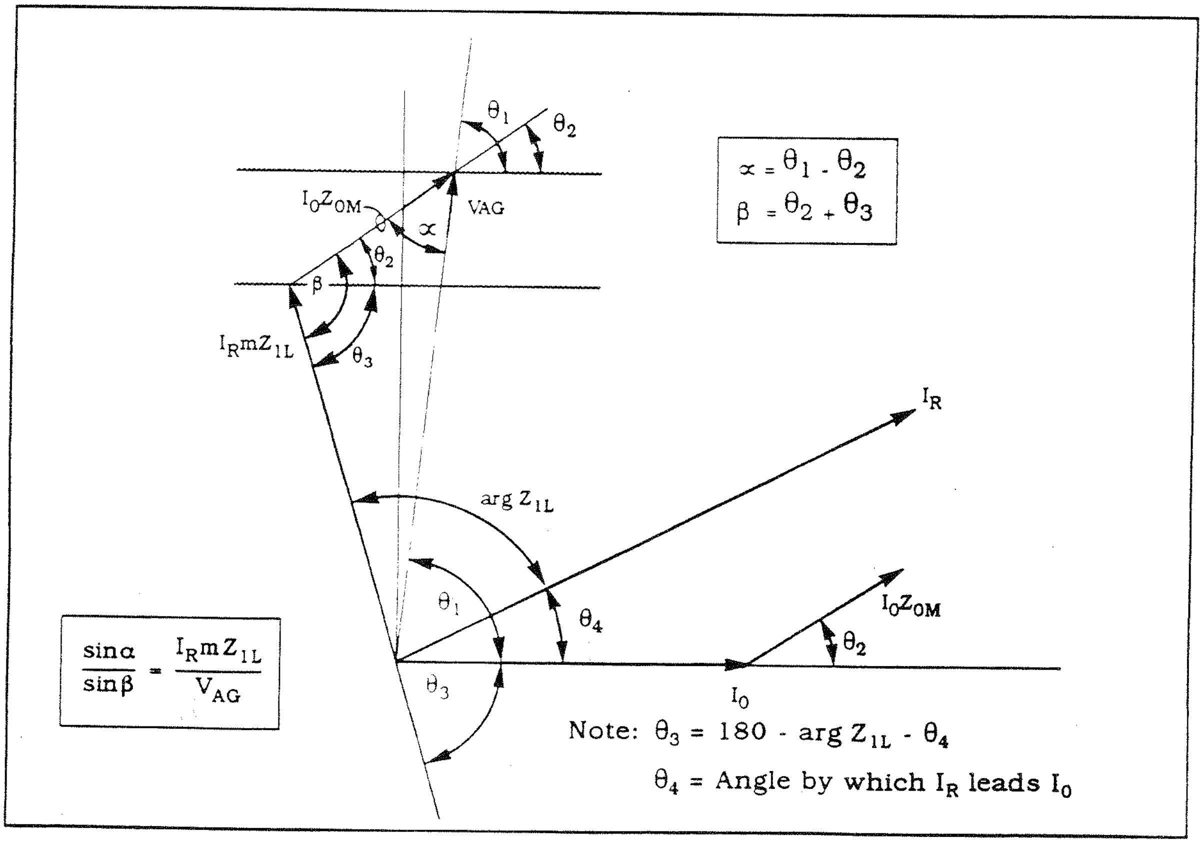


Figure 13. Determination of Angles for Use In "Law of Sines" for Fault Location for Phase-to-Ground Fault.

determine m with fault resistance present as was first suggested by J.M. Crockett of ABB Canada. Unfortunately m cannot be identified by this method with both fault resistance and mutual present.

2. The introduction of adjacent line zero sequence current into the evaluation may be helpful. However, the mutual influence is not only dependent on $I_{0E}$ and $Z_{0m}$ (the adjacent line zero sequence current and the mutual impedance) but also on m, the location of the fault. This concept may be helpful in defining the boundary of zone-1 reach, but is inadequate for pinpointing fault location.
3. The use of the imaginary component of the calculated ratio of $V_{\phi G}$ and $I_\phi + KI_0$ is useful for improving accuracy, where fault resistance is the significant error producer, but this affords little benefit where zero sequence mutual is the primary cause of error.
4. Takagi [5] suggested a method to minimize the effect of fault resistance in the distance determination process which utilizes the imaginary part of the product of voltage and the conjugate of fault current (with pre fault load current removed) divided by the imaginary part of the product of impedance per unit length, fault current and the conjugate of fault current (with load removed). This method provides some benefit in determining from locally available quantities the distance to a fault having fault resistance and out-of-phase sources, but offers little or no help in reducing the error associated with zero sequence mutual.

## Two Ended Evaluation

None of the single ended evaluation schemes described, provide adequate elimination of the errors associated with load, fault resistance and zero sequence mutual. However, very simple procedures can be utilized by hand or through available software with the data preserved by modern microprocessor relays from both terminals to identify fault location without being subject to these errors. The method uses negative sequence quantities for unbalanced faults and positive sequence quantities for 3-phase faults. Figure 14 shows the method by which an unbalanced fault is located using negative

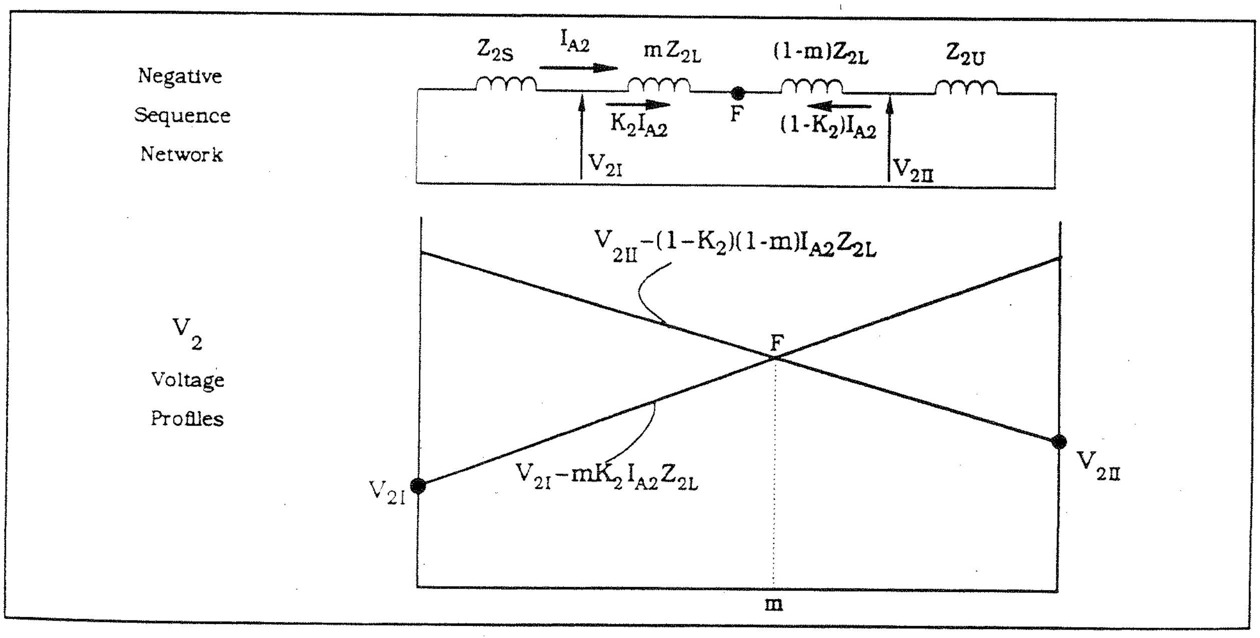


Figure 14. Negative Sequence Voltage Profiles Versus Per Unit Distance *m* for fault at F.

sequence voltage and current measured at each terminal. Using this method on a two terminal line where there are no tapped loads or third terminals with sources, excellent accuracy can be achieved.

## Fault Generated Signals

Using ground fault generated signals is also an effective method of identifying fault location. It requires a receiver at each location to recognize the initial change resulting from the fault. The signal is conducted at each terminal through a voltage transformer or a coupling capacitor. It is then retransmitted from one terminal through microwave to the far terminal, where it is compared with the signal received by the direct path. By proper compensation of the channel delay, the time of arrival of two signals may be compared to identify fault location. Aside from the difficulties associated with the channel itself, and varying propagation velocities on the transmission line, reasonable accuracy should be possible, devoid of the complications of pre-fault load, fault resistance, and zero sequence mutual effects.

## Conclusions

Identifying ground fault location for relaying or measurement is a complicated process. Only in the simplest of circumstances can a relaying system, equipped with fault locating provision, provide a precise estimate of location using the information at one transmission line terminal. However, using a communications medium to convey fault data to a common location allows a precise determination to be made.

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